**CONSUMABLE WORKBOOKS** Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks in both English and Spanish.

<table>
<thead>
<tr>
<th>MHID</th>
<th>ISBN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Study Guide and Intervention Workbook</td>
<td>0-07-660301-6 978-0-07-660301-5</td>
</tr>
<tr>
<td>Homework Practice Workbook</td>
<td>0-07-660299-0 978-0-07-660299-5</td>
</tr>
<tr>
<td>Spanish Version</td>
<td></td>
</tr>
<tr>
<td>Homework Practice Workbook</td>
<td>0-07-660300-8 978-0-07-660300-8</td>
</tr>
</tbody>
</table>

**Answers For Workbooks** The answers for Chapter 4 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

**ConnectED** All of the materials found in this booklet are included for viewing, printing, and editing at connected.mcgraw-hill.com.

**Spanish Assessment Masters** (MHID: 0-07-660298-2, ISBN: 978-0-07-660298-8) These masters contain a Spanish version of Chapter 4 Test Form 2A and Form 2C.

---

connected.mcgraw-hill.com
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Teacher’s Guide to Using the
Chapter 4 Resource Masters

The Chapter 4 Resource Masters includes the core materials needed for Chapter 4. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing, printing, and editing at connectED.mcgraw-hill.com.

Chapter Resources

**Student-Built Glossary** (pages 1–2)
These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Give this to students before beginning Lesson 4-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

**Anticipation Guide** (pages 3–4) This master, presented in both English and Spanish, is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

**Study Guide and Intervention** These masters provide vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

**Skills Practice** This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

**Practice** This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

**Word Problem Practice** This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

**Enrichment** These activities may extend the concepts of the lesson, offer an historical or multicultural look at the concepts, or widen students’ perspectives on the mathematics they are learning. They are written for use with all levels of students.

**Graphing Calculator, TI-Nspire, or Spreadsheet Activities** These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.
Assessment Options

The assessment masters in the Chapter 4 Resource Masters offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

Student Recording Sheet This master corresponds with the standardized test practice at the end of the chapter.

Extended Response Rubric This master provides information for teachers and students on how to assess performance on open-ended questions.

Quizzes Four free-response quizzes offer assessment at appropriate intervals in the chapter.

Mid-Chapter Test This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

Vocabulary Test This test is suitable for all students. It includes a list of vocabulary words and 10 questions to assess students’ knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

Leveled Chapter Tests

- Form 1 contains multiple-choice questions and is intended for use with below grade level students.
- Forms 2A and 2B contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- Forms 2C and 2D contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- Form 3 is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

Extended-Response Test Performance assessment tasks are suitable for all students. Sample answers and a scoring rubric are included for evaluation.

Standardized Test Practice These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

Answers

- The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages.
- Full-size answer keys are provided for the assessment masters.
This is an alphabetical list of the key vocabulary terms you will learn in Chapter 4. As you study the chapter, complete each term’s definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition/Description/Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>completing the square</td>
<td></td>
<td></td>
</tr>
<tr>
<td>complex conjugates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>complex number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>constant term</td>
<td></td>
<td></td>
</tr>
<tr>
<td>discriminant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(dihs-KRIH-muh-nuhnt)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>imaginary unit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>linear term</td>
<td></td>
<td></td>
</tr>
<tr>
<td>maximum value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>minimum value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>pure imaginary number</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(continued on the next page)
<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition/Description/Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>quadratic equation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(kwah-DRA-thk)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quadratic Formula</td>
<td></td>
<td></td>
</tr>
<tr>
<td>quadratic inequality</td>
<td></td>
<td></td>
</tr>
<tr>
<td>quadratic term</td>
<td></td>
<td></td>
</tr>
<tr>
<td>root</td>
<td></td>
<td></td>
</tr>
<tr>
<td>standard form</td>
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<td></td>
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<tr>
<td>vertex</td>
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</tr>
<tr>
<td>vertex form</td>
<td></td>
<td></td>
</tr>
<tr>
<td>zero</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4 Anticipation Guide

Quadratic Functions and Relations

Step 1 Before you begin Chapter 4

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

<table>
<thead>
<tr>
<th>STEP 1 A, D, or NS</th>
<th>Statement</th>
<th>STEP 2 A or D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. A</td>
<td>All quadratic functions have a term with the variable to the second power.</td>
<td></td>
</tr>
<tr>
<td>2. D</td>
<td>If the graph of the quadratic function $y = ax^2 + c$ opens up then $c &lt; 0$.</td>
<td></td>
</tr>
<tr>
<td>3. A</td>
<td>A quadratic equation whose graph does not intersect the $x$-axis has no real solution.</td>
<td></td>
</tr>
<tr>
<td>4. D</td>
<td>Since graphing shows the exact solutions to a quadratic equation, no other method is necessary for solving.</td>
<td></td>
</tr>
<tr>
<td>5. A</td>
<td>If $(x - 3)(x + 4) = 0$, then either $x - 3 = 0$ or $x + 4 = 0$.</td>
<td></td>
</tr>
<tr>
<td>6. D</td>
<td>An imaginary number contains $i$, which equals the square root of $-1$.</td>
<td></td>
</tr>
<tr>
<td>7. A</td>
<td>A method called completing the square can be used to rewrite a quadratic expression as a perfect square.</td>
<td></td>
</tr>
<tr>
<td>8. D</td>
<td>The quadratic formula can only be used for quadratic equations that cannot be solved by graphing or completing the square.</td>
<td></td>
</tr>
<tr>
<td>9. A</td>
<td>The discriminant of a quadratic equation can be used to determine the direction the graph will open.</td>
<td></td>
</tr>
<tr>
<td>10. A</td>
<td>The graph of $y = 2x^2$ is a dilation of the graph of $y = x^2$.</td>
<td></td>
</tr>
<tr>
<td>11. D</td>
<td>The graph of $y = (x + 2)^2$ will be two units to the right of the graph of $y = x^2$.</td>
<td></td>
</tr>
<tr>
<td>12. A</td>
<td>The graph of a quadratic inequality containing the symbol $&lt;$ will be a parabola opening downward.</td>
<td></td>
</tr>
</tbody>
</table>

Step 2 After you complete Chapter 4

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.
## Pasos 1

### Antes de comenzar el Capítulo 4

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a)).

<table>
<thead>
<tr>
<th>PASO 1 A, D o NS</th>
<th>Enunciado</th>
<th>PASO 2 A o D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Todas las funciones cuadráticas tienen un término con la variable elevada a la segunda potencia.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Si la gráfica de la función cuadrática $y = ax^2 + c$ se abre hacia arriba, entonces $c &gt; 0$.</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Una ecuación cuadrática cuya gráfica no interseca el eje x no tiene solución real.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Dado que graficar muestra las soluciones exactas de una ecuación cuadrática, no se necesita ningún otro método para resolverla.</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Si $(x - 3)(x + 4) = 0$, entonces $x - 3 = 0$ ó $x + 4 = 0$.</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>Un número imaginario contiene $i$, la cual es igual a la raíz cuadrada de $-1$.</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>El método de completar el cuadrado se puede usar para volver a plantear una expresión cuadrática como un cuadrado perfecto.</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>La fórmula cuadrática se puede usar sólo para ecuaciones cuadráticas que no pueden resolverse mediante la completación del cuadrado o una gráfica.</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>Se puede usar el discriminante de una ecuación cuadrática para determinar la dirección en que se abrirá la gráfica.</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>La gráfica de $y = 2x^2$ es una dilatación de la gráfica de $y = x^2$.</td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>La gráfica de $y = (x + 2)^2$ estará dos unidades a la derecha de la gráfica de $y = x^2$.</td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td>La gráfica de una desigualdad cuadrática con el símbolo $&lt;$ será una parábola que se abre hacia abajo.</td>
<td></td>
</tr>
</tbody>
</table>

### Pasos 2

### Después de completar el Capítulo 4

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.
4-1 Study Guide and Intervention

Graphing Quadratic Functions

**Graph Quadratic Functions**

**Quadratic Function**

A function defined by an equation of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$

**Graph of a Quadratic Function**

A parabola with these characteristics:

- y-intercept: $c$;
- axis of symmetry: $x = \frac{-b}{2a}$;
- x-coordinate of vertex: $\frac{-b}{2a}$

---

**Example** Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex for the graph of $f(x) = x^2 - 3x + 5$. Use this information to graph the function.

$a = 1, b = -3, c = 5$, so the y-intercept is 5. The equation of the axis of symmetry is $x = \frac{-(-3)}{2(1)}$ or $x = \frac{3}{2}$. The x-coordinate of the vertex is $\frac{3}{2}$.

Next make a table of values for $x$ near $\frac{3}{2}$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$x^2 - 3x + 5$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$0^2 - 3(0) + 5$</td>
<td>5</td>
<td>(0, 5)</td>
</tr>
<tr>
<td>1</td>
<td>$1^2 - 3(1) + 5$</td>
<td>3</td>
<td>(1, 3)</td>
</tr>
<tr>
<td>$\frac{3}{2}$</td>
<td>$(\frac{3}{2})^2 - 3(\frac{3}{2}) + 5$</td>
<td>$\frac{11}{4}$</td>
<td>$(\frac{3}{2}, \frac{11}{4})$</td>
</tr>
<tr>
<td>2</td>
<td>$2^2 - 3(2) + 5$</td>
<td>3</td>
<td>(2, 3)</td>
</tr>
<tr>
<td>3</td>
<td>$3^2 - 3(3) + 5$</td>
<td>5</td>
<td>(3, 5)</td>
</tr>
</tbody>
</table>

**Exercises**

Complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

1. $f(x) = x^2 + 6x + 8$
2. $f(x) = -x^2 - 2x + 2$
3. $f(x) = 2x^2 - 4x + 3$
Graphing Quadratic Functions

Maximum and Minimum Values The y-coordinate of the vertex of a quadratic function is the maximum value or minimum value of the function.

| Maximum or Minimum Value of a Quadratic Function | The graph of \( f(x) = ax^2 + bx + c \), where \( a \neq 0 \), opens up and has a minimum when \( a > 0 \). The graph opens down and has a maximum when \( a < 0 \). |

Example Determine whether each function has a maximum or minimum value, and find that value. Then state the domain and range of the function.

a. \( f(x) = 3x^2 - 6x + 7 \)

For this function, \( a = 3 \) and \( b = -6 \).

Since \( a > 0 \), the graph opens up, and the function has a minimum value.

The minimum value is the y-coordinate of the vertex. The x-coordinate of the vertex is \( \frac{-b}{2a} = \frac{-(-6)}{2(3)} = 1 \).

Evaluate the function at \( x = 1 \) to find the minimum value.

\[ f(1) = 3(1)^2 - 6(1) + 7 = 4 \]

so the minimum value of the function is 4. The domain is all real numbers. The range is all reals greater than or equal to the minimum value, that is \( \{f(x) | f(x) \geq 4\} \).

b. \( f(x) = 100 - 2x - x^2 \)

For this function, \( a = -1 \) and \( b = -2 \).

Since \( a < 0 \), the graph opens down, and the function has a maximum value.

The maximum value is the y-coordinate of the vertex. The x-coordinate of the vertex is \( \frac{-b}{2a} = \frac{-(-2)}{2(-1)} = -1 \).

Evaluate the function at \( x = -1 \) to find the maximum value.

\[ f(-1) = 100 - 2(-1) - (-1)^2 = 101 \]

so the maximum value of the function is 101. The domain is all real numbers. The range is all reals less than or equal to the maximum value, that is \( \{f(x) | f(x) \leq 101\} \).

Exercises Determine whether each function has a maximum or minimum value, and find that value. Then state the domain and range of the function.

1. \( f(x) = 2x^2 - x + 10 \)
2. \( f(x) = x^2 + 4x - 7 \)
3. \( f(x) = 3x^2 - 3x + 1 \)

4. \( f(x) = x^2 + 5x + 2 \)
5. \( f(x) = 20 + 6x - x^2 \)
6. \( f(x) = 4x^2 + x + 3 \)

7. \( f(x) = -x^2 - 4x + 10 \)
8. \( f(x) = x^2 - 10x + 5 \)
9. \( f(x) = -6x^2 + 12x + 21 \)
4-1 Skills Practice

Graphing Quadratic Functions

Complete parts a–c for each quadratic function.

a. Find the $y$-intercept, the equation of the axis of symmetry, and the $x$-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

1. $f(x) = -2x^2$
2. $f(x) = x^2 - 4x + 4$
3. $f(x) = x^2 - 6x + 8$

Determine whether each function has a maximum or a minimum value, and find that value. Then state the domain and range of the function.

4. $f(x) = 6x^2$
5. $f(x) = -8x^2$
6. $f(x) = x^2 + 2x$

7. $f(x) = -2x^2 + 4x - 3$
8. $f(x) = 3x^2 + 12x + 3$
9. $f(x) = 2x^2 + 4x + 1$

10. $f(x) = 3x^2$
11. $f(x) = x^2 + 1$
12. $f(x) = -x^2 + 6x - 15$

13. $f(x) = 2x^2 - 11$
14. $f(x) = x^2 - 10x + 5$
15. $f(x) = -2x^2 + 8x + 7$
4-1 Practice

Graphing Quadratic Functions

Complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

1. \( f(x) = x^2 - 8x + 15 \)
2. \( f(x) = -x^2 - 4x + 12 \)
3. \( f(x) = 2x^2 - 2x + 1 \)

Determine whether each function has a maximum or minimum value, and find that value. Then state the domain and range of the function.

4. \( f(x) = x^2 + 2x - 8 \)
5. \( f(x) = x^2 - 6x + 14 \)
6. \( v(x) = -x^2 + 14x - 57 \)

7. \( f(x) = 2x^2 + 4x - 6 \)
8. \( f(x) = -x^2 + 4x - 1 \)
9. \( f(x) = -\frac{2}{3}x^2 + 8x - 24 \)

10. GRAVITATION From 4 feet above a swimming pool, Susan throws a ball upward with a velocity of 32 feet per second. The height \( h(t) \) of the ball \( t \) seconds after Susan throws it is given by \( h(t) = -16t^2 + 32t + 4 \). For \( t \geq 0 \), find the maximum height reached by the ball and the time that this height is reached.

11. HEALTH CLUBS Last year, the SportsTime Athletic Club charged $20 to participate in an aerobics class. Seventy people attended the classes. The club wants to increase the class price this year. They expect to lose one customer for each $1 increase in the price.

a. What price should the club charge to maximize the income from the aerobics classes?

b. What is the maximum income the SportsTime Athletic Club can expect to make?
1. TRAJECTORIES A cannonball is launched from a cannon on the wall of Fort Chambly, Quebec. If the path of the cannonball is traced on a piece of graph paper aligned so that the cannon is situated on the y-axis, the equation that describes the path is

\[ y = -\frac{1}{1600}x^2 + \frac{1}{2}x + 20, \]

where \( x \) is the horizontal distance from the cliff and \( y \) is the vertical distance above the ground in feet. How high above the ground is the cannon?

2. TICKETING The manager of a symphony computes that the symphony will earn \(-40P^2 + 1100P\) dollars per concert if they charge \( P \) dollars for tickets. What ticket price should the symphony charge in order to maximize its profits?

3. ARCHES An architect decides to use a parabolic arch for the main entrance of a science museum. In one of his plans, the top edge of the arch is described by the graph of \( y = -\frac{1}{4}x^2 + \frac{5}{2}x + 15 \). What are the coordinates of the vertex of this parabola?

4. FRAMING A frame company offers a line of square frames. If the side length of the frame is \( s \), then the area of the opening in the frame is given by the function \( a(s) = s^2 - 10s + 24 \). Graph \( a(s) \).

5. WALKING Canal Street and Walker Street are perpendicular to each other. Evita is driving south on Canal Street and is currently 5 miles north of the intersection with Walker Street. Jack is at the intersection of Canal and Walker Streets and heading east on Walker. Jack and Evita are both driving 30 miles per hour.

   a. When Jack is \( x \) miles east of the intersection, where is Evita?

   b. The distance between Jack and Evita is given by the formula \( \sqrt{x^2 + (5 - x)^2} \). For what value of \( x \) are Jack and Evita at their closest? (Hint: Minimize the square of the distance.)

   c. What is the distance of closest approach?
**Finding the x-intercepts of a Parabola**

As you know, if \( f(x) = ax^2 + bx + c \) is a quadratic function, the values of \( x \) that make \( f(x) \) equal to zero are \( \frac{-b + \sqrt{b^2 - 4ac}}{2a} \) and \( \frac{-b - \sqrt{b^2 - 4ac}}{2a} \).

The average of these two number values is \( -\frac{b}{2a} \).

The function \( f(x) \) has its maximum or minimum value when \( x = -\frac{b}{2a} \). The x-intercepts of the parabola, when they exist, are \( \frac{\sqrt{b^2 - 4ac}}{2a} \) units to the left and right of the axis of symmetry.

**Example**

Find the vertex, axis of symmetry, and x-intercepts for \( f(x) = 5x^2 + 10x - 7 \).

Use \( x = -\frac{b}{2a} \).

\[
x = -\frac{10}{2(5)} = -1 
\]

The x-coordinate of the vertex is \(-1\).

Substitute \( x = -1 \) in \( f(x) = 5x^2 + 10x - 7 \).

\[
f(-1) = 5(-1)^2 + 10(-1) - 7 = -12.
\]

The vertex is \((-1, -12)\).

The axis of symmetry is \( x = -\frac{b}{2a} \), or \( x = -1 \).

The x-coordinates of the x-intercepts are \( -1 \pm \frac{\sqrt{b^2 - 4ac}}{2a} \) and \( -1 \pm \frac{\sqrt{240}}{10} \). The x-intercepts are \( \left(-1 - \frac{2}{5}\sqrt{15}, 0\right) \) and \( \left(-1 + \frac{2}{5}\sqrt{15}, 0\right) \).

**Exercises**

Find the vertex, axis of symmetry, and x-intercepts for the graph of each function using \( x = -\frac{b}{2a} \).

1. \( f(x) = x^2 - 4x - 8 \)

2. \( g(x) = -4x^2 - 8x + 3 \)

3. \( y = -x^2 + 8x + 3 \)

4. \( f(x) = 2x^2 + 6x + 5 \)

5. \( A(x) = x^2 + 12x + 36 \)

6. \( k(x) = -2x^2 + 2x - 6 \)
SOLVING QUADRATIC EQUATIONS BY GRAPHING

**Solve Quadratic Equations**

<table>
<thead>
<tr>
<th>Quadratic Equation</th>
<th>A quadratic equation has the form $ax^2 + bx + c = 0$, where $a \neq 0$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roots of a Quadratic Equation</td>
<td>solution(s) of the equation, or the zero(s) of the related quadratic function</td>
</tr>
</tbody>
</table>

The zeros of a quadratic function are the $x$-intercepts of its graph. Therefore, finding the $x$-intercepts is one way of solving the related quadratic equation.

**Example**

**Solve $x^2 + x - 6 = 0$ by graphing.**

Graph the related function $f(x) = x^2 + x - 6$.

The $x$-coordinate of the vertex is $\frac{-b}{2a} = \frac{-1}{2}$, and the equation of the axis of symmetry is $x = \frac{-1}{2}$.

Make a table of values using $x$-values around $\frac{-1}{2}$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-1$</th>
<th>$-\frac{1}{2}$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>$-6$</td>
<td>$-\frac{1}{4}$</td>
<td>$-6$</td>
<td>$-4$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

From the table and the graph, we can see that the zeros of the function are $2$ and $-3$.

**Exercises**

Use the related graph of each equation to determine its solution.

1. $x^2 + 2x - 8 = 0$
2. $x^2 - 4x - 5 = 0$
3. $x^2 - 5x + 4 = 0$
4. $x^2 - 10x + 21 = 0$
5. $x^2 + 4x + 6 = 0$
6. $4x^2 + 4x + 1 = 0$
Estimate Solutions Often, you may not be able to find exact solutions to quadratic equations by graphing. But you can use the graph to estimate solutions.

Example Solve \( x^2 - 2x - 2 = 0 \) by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

The equation of the axis of symmetry of the related function is \( x = \frac{-2}{2(1)} = 1 \), so the vertex has \( x \)-coordinate 1. Make a table of values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>1</td>
<td>-2</td>
<td>-3</td>
<td>-2</td>
<td>1</td>
</tr>
</tbody>
</table>

The \( x \)-intercepts of the graph are between 2 and 3 and between 0 and \(-1\). So one solution is between 2 and 3, and the other solution is between 0 and \(-1\).

Exercises Solve the equations. If exact roots cannot be found, state the consecutive integers between which the roots are located.

1. \( x^2 - 4x + 2 = 0 \)
2. \( x^2 + 6x + 6 = 0 \)
3. \( x^2 + 4x + 2 = 0 \)

4. \( -x^2 + 2x + 4 = 0 \)
5. \( 2x^2 - 12x + 17 = 0 \)
6. \( -\frac{1}{2}x^2 + x + \frac{5}{2} = 0 \)
### 4-2 Skills Practice

**Solving Quadratic Equations By Graphing**

Use the related graph of each equation to determine its solutions.

1. \(x^2 + 2x - 3 = 0\)
2. \(-x^2 - 6x - 9 = 0\)
3. \(3x^2 + 4x + 3 = 0\)

![Graphs of equations 1, 2, and 3]

Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. \(x^2 - 6x + 5 = 0\)
5. \(-x^2 + 2x - 4 = 0\)
6. \(x^2 - 6x + 4 = 0\)

![Graphs of equations 4, 5, and 6]

7. \(-x^2 - 4x = 0\)
8. \(-x^2 + 36 = 0\)

![Graphs of equations 7 and 8]
### 4-2 Practice

**Solving Quadratic Equations By Graphing**

Use the related graph of each equation to determine its solutions.

1. \(-3x^2 + 3 = 0\)
2. \(3x^2 + x + 3 = 0\)
3. \(x^2 - 3x + 2 = 0\)

Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. \(-2x^2 - 6x + 5 = 0\)
5. \(x^2 + 10x + 24 = 0\)
6. \(2x^2 - x - 6 = 0\)

7. \(-x^2 + x + 6 = 0\)
8. \(-x^2 + 5x - 8 = 0\)

9. **GRAVITY** Use the formula \(h(t) = v_0t - 16t^2\), where \(h(t)\) is the height of an object in feet, \(v_0\) is the object’s initial velocity in feet per second, and \(t\) is the time in seconds.

   a. Marta throws a baseball with an initial upward velocity of 60 feet per second. Ignoring Marta’s height, how long after she releases the ball will it hit the ground?

   b. A volcanic eruption blasts a boulder upward with an initial velocity of 240 feet per second. How long will it take the boulder to hit the ground if it lands at the same elevation from which it was ejected?
1. **TRAJECTORIES** David threw a baseball into the air. The function of the height of the baseball in feet is \( h = 80t - 16t^2 \), where \( t \) represents the time in seconds after the ball was thrown. Use this graph of the function to determine how long it took for the ball to fall back to the ground.

![Graph of height vs time]

2. **BRIDGES** In 1895, a brick arch railway bridge was built on North Avenue in Baltimore, Maryland. The arch is described by the equation \( h = 9 - \frac{1}{50}x^2 \), where \( h \) is the height in yards and \( x \) is the distance in yards from the center of the bridge. Graph this equation and describe, to the nearest yard, where the bridge touches the ground.

![Graph of height vs distance]

3. **LOGIC** Wilma is thinking of two numbers. The sum is 2 and the product is -24. Use a quadratic equation to find the two numbers.

4. **RADIO TELESCOPES** The cross-section of a large radio telescope is a parabola. The dish is set into the ground. The equation that describes the cross-section is \( d = \frac{2}{75}x^2 - \frac{4}{3}x - \frac{32}{3} \), where \( d \) gives the depth of the dish below ground and \( x \) is the distance from the control center, both in meters. If the dish does not extend above the ground level, what is the diameter of the dish? Solve by graphing.

![Graph of depth vs distance]

5. **BOATS** The distance between two boats is \( d = \sqrt{t^2 - 10t + 35} \), where \( d \) is distance in meters and \( t \) is time in seconds.

   a. Make a graph of \( d^2 \) versus \( t \).

   ![Graph of distance squared vs time]

   b. Do the boats ever collide?
Graphing Absolute Value Equations

You can solve absolute value equations in much the same way you solved quadratic equations. Graph the related absolute value function for each equation using a graphing calculator. Then use the ZERO feature in the CALC menu to find its real solutions, if any. Recall that solutions are points where the graph intersects the x-axis.

For each equation, make a sketch of the related graph and find the solutions rounded to the nearest hundredth.

1. \(|x + 5| = 0\)
2. \(|4x - 3| + 5 = 0\)
3. \(|x - 7| = 0\)
4. \(|x + 3| - 8 = 0\)
5. \(-|x + 3| + 6 = 0\)
6. \(|x - 2| - 3 = 0\)
7. \(|3x + 4| = 2\)
8. \(|x + 12| = 10\)
9. \(|x| - 3 = 0\)

10. Explain how solving absolute value equations algebraically and finding zeros of absolute value functions graphically are related.
Factored Form To write a quadratic equation with roots \( p \) and \( q \), let \((x - p)(x - q) = 0\). Then multiply using FOIL.

Example Write a quadratic equation in standard form with the given roots.

a. 3, -5

\[(x - p)(x - q) = 0\] Write the pattern.
\[(x - 3)(x - (-5)) = 0\] Replace \( p \) with 3, \( q \) with -5.
\[(x - 3)(x + 5) = 0\] Simplify.
\[x^2 + 2x - 15 = 0\] Use FOIL.

The equation \( x^2 + 2x - 15 = 0 \) has roots 3 and -5.

b. \(-\frac{7}{8}, \frac{1}{3}\)

\[(x - p)(x - q) = 0\]
\[\left(x - \left(-\frac{7}{8}\right)\right)\left(x - \frac{1}{3}\right) = 0\]
\[\left(x + \frac{7}{8}\right)\left(x - \frac{1}{3}\right) = 0\]
\[\frac{24 \cdot (8x + 7)(3x - 1)}{24} = 24 \cdot 0\]
\[24x^2 + 13x - 7 = 0\]

The equation \( 24x^2 + 13x - 7 = 0 \) has roots \(-\frac{7}{8}\) and \(\frac{1}{3}\).

Exercises

Write a quadratic equation in standard form with the given root(s).

1. 3, -4
2. -8, -2
3. 1, 9
4. -5
5. 10, 7
6. -2, 15
7. \(-\frac{1}{3}, 5\)
8. 2, \(\frac{2}{3}\)
9. \(-7, \frac{3}{4}\)
10. 3, \(\frac{2}{5}\)
11. \(-\frac{4}{9}, -1\)
12. 9, \(\frac{1}{6}\)
13. \(\frac{2}{3}, -\frac{2}{3}\)
14. \(\frac{5}{4}, -\frac{1}{2}\)
15. \(\frac{3}{7}, \frac{1}{5}\)
16. \(-\frac{7}{8}, \frac{7}{2}\)
17. \(\frac{1}{2}, \frac{3}{4}\)
18. \(\frac{1}{8}, \frac{1}{6}\)
Study Guide and Intervention (continued)

Solving Quadratic Equations by Factoring

Solve Equations by Factoring
When you use factoring to solve a quadratic equation, you use the following property.

Zero Product Property
For any real numbers $a$ and $b$, if $ab = 0$, then either $a = 0$ or $b = 0$, or both $a$ and $b = 0$.

Example
Solve each equation by factoring.

a. $3x^2 = 15x$
   
   $3x^2 = 15x$  Original equation  
   $3x^2 - 15x = 0$  Subtract $15x$ from both sides.  
   $3x(x - 5) = 0$  Factor the binomial.  
   $3x = 0$ or $x - 5 = 0$  Zero Product Property  
   $x = 0$ or $x = 5$  Solve each equation.  

   The solution set is \{0, 5\}.

b. $4x^2 - 5x = 21$
   
   $4x^2 - 5x - 21 = 0$  Original equation  
   $(4x + 7)(x - 3) = 0$  Factor the trinomial.  
   $4x + 7 = 0$ or $x - 3 = 0$  Zero Product Property  
   $x = -\frac{7}{4}$ or $x = 3$  Solve each equation.  

   The solution set is \{-\frac{7}{4}, 3\}.

Exercises
Solve each equation by factoring.

1. $6x^2 - 2x = 0$  
2. $x^2 = 7x$  
3. $20x^2 = -25x$

4. $6x^2 = 7x$  
5. $6x^2 - 27x = 0$  
6. $12x^2 - 8x = 0$

7. $x^2 + x - 30 = 0$  
8. $2x^2 - x - 3 = 0$  
9. $x^2 + 14x + 33 = 0$

10. $4x^2 + 27x - 7 = 0$  
11. $3x^2 + 29x - 10 = 0$  
12. $6x^2 - 5x - 4 = 0$

13. $12x^2 - 8x + 1 = 0$  
14. $5x^2 + 28x - 12 = 0$  
15. $2x^2 - 250x + 5000 = 0$

16. $2x^2 - 11x - 40 = 0$  
17. $2x^2 + 21x - 11 = 0$  
18. $3x^2 + 2x - 21 = 0$

19. $8x^2 - 14x + 3 = 0$  
20. $6x^2 + 11x - 2 = 0$  
21. $5x^2 + 17x - 12 = 0$

22. $12x^2 + 25x + 12 = 0$  
23. $12x^2 + 18x + 6 = 0$  
24. $7x^2 - 36x + 5 = 0$
**4-3 Skills Practice**

**Solving Quadratic Equations by Factoring**

Write a quadratic equation in standard form with the given root(s).

1. 1, 4
2. 6, −9

3. −2, −5
4. 0, 7

5. −\(1/3\), −3
6. −\(1/2\), \(3/4\)

Factor each polynomial.

7. \(m^2 + 7m - 18\)
8. \(2x^2 - 3x - 5\)

9. \(4x^2 + 4x - 15\)
10. \(4p^2 + 4p - 24\)

11. \(3y^2 + 21y + 36\)
12. \(c^2 - 100\)

Solve each equation by factoring.

13. \(x^2 = 64\)
14. \(x^2 - 100 = 0\)

15. \(x^2 - 3x + 2 = 0\)
16. \(x^2 - 4x + 3 = 0\)

17. \(x^2 + 2x - 3 = 0\)
18. \(x^2 - 3x - 10 = 0\)

19. \(x^2 - 6x + 5 = 0\)
20. \(x^2 - 9x = 0\)

21. \(x^2 - 4x = 21\)
22. \(2x^2 + 5x - 3 = 0\)

23. \(4x^2 + 5x - 6 = 0\)
24. \(3x^2 - 13x - 10 = 0\)

25. **NUMBER THEORY** Find two consecutive integers whose product is 272.
4-3 Practice

Solving Quadratic Equations by Factoring

Write a quadratic equation in standard form with the given root(s).

1. 7, 2  
2. 0, 3  
3. −5, 8

4. −7, −8  
5. −6, −3  
6. 3, −4

7. 1, \frac{1}{2}  
8. \frac{1}{3}, 2  
9. 0, −\frac{7}{2}

Factor each polynomial.

10. r^3 + 3r^2 − 54r  
11. 8a^2 + 2a − 6  
12. c^2 − 49

13. x^3 + 8  
14. 16r^2 − 169  
15. b^4 − 81

Solve each equation by factoring.

16. x^2 − 4x − 12 = 0  
17. x^2 − 16x + 64 = 0

18. x^2 − 6x + 8 = 0  
19. x^2 + 3x + 2 = 0

20. x^2 − 4x = 0  
21. 7x^2 = 4x

22. 10x^2 = 9x  
23. x^2 = 2x + 99

24. x^2 + 12x = −36  
25. 5x^2 − 35x + 60 = 0

26. 36x^2 = 25  
27. 2x^2 − 8x − 90 = 0

28. NUMBER THEORY Find two consecutive even positive integers whose product is 624.

29. NUMBER THEORY Find two consecutive odd positive integers whose product is 323.

30. GEOMETRY The length of a rectangle is 2 feet more than its width. Find the dimensions of the rectangle if its area is 63 square feet.

31. PHOTOGRAPHY The length and width of a 6-inch by 8-inch photograph are reduced by the same amount to make a new photograph whose area is half that of the original. By how many inches will the dimensions of the photograph have to be reduced?
1. **FLASHLIGHTS** When Dora shines her flashlight on the wall at a certain angle, the edge of the lit area is in the shape of a parabola. The equation of the parabola is \( y = 2x^2 + 2x - 60 \). Factor this quadratic equation.

2. **SIGNS** David was looking through an old algebra book and came across this equation.

\[
x^2 + 6x + 8 = 0
\]

The sign in front of the 6 was blotted out. How does the missing sign depend on the signs of the roots?

3. **ART** The area in square inches of the drawing *Maisons près de la mer* by Claude Monet is approximated by the equation \( y = x^2 - 23x + 130 \). Factor the equation to find the two roots, which are equal to the approximate length and width of the drawing.

4. **PROGRAMMING** Ray is a computer programmer. He needs to find the quadratic function of this graph for an algorithm related to a game involving dice. Provide such a function.

5. **ANIMATION** A computer graphics animator would like to make a realistic simulation of a tossed ball. The animator wants the ball to follow the parabolic trajectory represented by the quadratic equation \( f(x) = -0.2(x + 5)(x - 5) \).

   a. What are the solutions of \( f(x) = 0 \)?

   b. Write \( f(x) \) in standard form.

   c. If the animator changes the equation to \( f(x) = -0.2x^2 + 20 \), what are the solutions of \( f(x) = 0 \)?
Enrichment

Using Patterns to Factor

Study the patterns below for factoring the sum and the difference of cubes.

\[a^3 + b^3 = (a + b)(a^2 - ab + b^2)\]
\[a^3 - b^3 = (a - b)(a^2 + ab + b^2)\]

This pattern can be extended to other odd powers. Study these examples.

**Example 1**  Factor \(a^5 + b^5\).

Extend the first pattern to obtain \(a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)\).

**Check:**

\[
(a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4) = a^5 - a^4b + a^3b^2 - a^2b^3 + ab^4 + a^4b - a^3b^2 + a^2b^3 - ab^4 + b^5
\]

\[= a^5 + b^5\]

**Example 2**  Factor \(a^5 - b^5\).

Extend the second pattern to obtain \(a^5 - b^5 = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)\).

**Check:**

\[
(a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4) = a^5 - a^4b - a^3b^2 - a^2b^3 - ab^4 - b^5
\]

In general, if \(n\) is an odd integer, when you factor \(a^n + b^n\) or \(a^n - b^n\), one factor will be either \((a + b)\) or \((a - b)\), depending on the sign of the original expression. The other factor will have the following properties:

- The first term will be \(a^{n-1}\) and the last term will be \(b^{n-1}\).
- The exponents of \(a\) will decrease by 1 as you go from left to right.
- The exponents of \(b\) will increase by 1 as you go from left to right.
- The degree of each term will be \(n - 1\).
- If the original expression was \(a^n + b^n\), the terms will alternately have + and – signs.
- If the original expression was \(a^n - b^n\), the terms will all have + signs.

**Use the patterns above to factor each expression.**

1. \(a^7 + b^7\)
2. \(c^9 - d^9\)
3. \(f^{11} + g^{11}\)

To factor \(x^{10} - y^{10}\), change it to \((x^5 + y^5)(x^5 - y^5)\) and factor each binomial. Use this approach to factor each expression.

4. \(x^{10} - y^{10}\)
5. \(a^{14} - b^{14}\)
Graphing Calculator Activity

Using Tables to Factor by Grouping

The **TABLE** feature of a graphing calculator can be used to help factor a polynomial of the form \(ax^2 + bx + c\). (The same problems can be solved with the **Lists and Spreadsheet** application on the **TI-Nspire**.)

**Example 1**

**Factor** \(10x^2 - 43x + 28\) **by grouping**.

Make a table of the negative factors of 10 or 280. Look for a pair of factors whose sum is \(-43\).

Enter the equation \(y = \frac{280}{x}\) in **Y1** to find the factors of 280. Then, find the sum of the factors using \(y = \frac{280}{x} + x\) in **Y2**. Set up the table to display the negative factors of 280 by setting \(\Delta Tbl = -1\). Examine the results.

Keystrokes:

```
Y= 280 ÷ X,T,θ,n ENTER VARS ENTER ENTER +
X,T,θ,n ENTER 2nd [TBLSET] (-1) ENTER (-1) ENTER 2nd [TABLE].
```

The last line of the table shows that \(-43x\) may be replaced with \(-8x + (-35x)\).

\[
10x^2 - 43x + 28 = 10x^2 - 8x + (-35x) + 28 \\
= 2x(5x - 4) + (-7)(5x - 4) \\
= (5x - 4)(2x - 7)
\]

Thus, \(10x^2 - 43x + 28\) = \((5x - 4)(2x - 7)\).

**Example 2**

**Factor** \(12x^2 - 7x - 12\).

Look at the factors of 12 or \(-144\) for a pair with a sum of \(-7\).

Enter an equation to determine the factors in **Y1** and an equation to find the sum of factors in **Y2**. Examine the table to find a sum of \(-7\).

Keystrokes:

```
Y=(-) 144 ÷ X,T,θ,n ENTER VARS ENTER ENTER +
X,T,θ,n ENTER 2nd [TBLSET] 1 ENTER 1 ENTER 2nd [TABLE].
```

\[
12x^2 - 7x - 12 = 12x^2 + 9x + (-16x) - 12 \\
= 3(4x^2 + 3x) - 4(4x + 3) \\
= (4x + 3)(3x - 4)
\]

Thus, \(12x^2 - 7x - 12\) = \((4x + 3)(3x - 4)\).

**Exercises**

Factor each polynomial.

1. \(y^2 - 20y - 96\)  
2. \(4z^2 - 33z + 35\)  
3. \(4y^2 + y - 18\)  
4. \(6a^2 + 2a - 15\)  
5. \(6m^2 + 17m + 12\)  
6. \(24z^2 - 46z + 15\)  
7. \(36y^2 + 84y + 49\)  
8. \(4b^2 + 36b - 403\)
Complex Numbers

Pure Imaginary Numbers A square root of a number \( n \) is a number whose square is \( n \). For nonnegative real numbers \( a \) and \( b \), \( \sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \) and \( \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \) \( b \neq 0 \).

- The imaginary unit \( i \) is defined to have the property that \( i^2 = -1 \).
- Simplified square root expressions do not have radicals in the denominator, and any number remaining under the square root has no perfect square factor other than 1.

Example 1

a. Simplify \( \sqrt{-48} \).

\[
\sqrt{-48} = \sqrt{16 \cdot (-3)} \\
\quad = \sqrt{16} \cdot \sqrt{3} \cdot \sqrt{-1} \\
\quad = 4i\sqrt{3}
\]

b. Simplify \( \sqrt{-63} \).

\[
\sqrt{-63} = \sqrt{-1} \cdot 7 \cdot 9 \\
\quad = \sqrt{-1} \cdot \sqrt{7} \cdot \sqrt{9} \\
\quad = 3i\sqrt{7}
\]

Example 2

a. Simplify \(-3i \cdot 4i\).

\[
-3i \cdot 4i = -12i^2 \\
\quad = -12(-1) \\
\quad = 12
\]

b. Simplify \( \sqrt{-3} \cdot \sqrt{-15} \).

\[
\sqrt{-3} \cdot \sqrt{-15} = i\sqrt{3} \cdot i\sqrt{15} \\
\quad = i^2\sqrt{45} \\
\quad = -1 \cdot \sqrt{9} \cdot \sqrt{5} \\
\quad = -3\sqrt{5}
\]

Example 3

Solve \( x^2 + 5 = 0 \).

\[
x^2 + 5 = 0 \\
\quad \rightarrow x^2 = -5 \\
\quad \rightarrow x = \pm \sqrt{5}i
\]

Exercises

Simplify.

1. \( \sqrt{-72} \) 

2. \( \sqrt{-24} \)

3. \( \sqrt{-84} \) 

4. \((2 + i)(2 - i)\)

Solve each equation.

5. \( 5x^2 + 45 = 0 \) 

6. \( 4x^2 + 24 = 0 \)

7. \( -9x^2 = 9 \) 

8. \( 7x^2 + 84 = 0 \)
4-4 Study Guide and Intervention (continued)

Complex Numbers

Operations with Complex Numbers

<table>
<thead>
<tr>
<th>Complex Number</th>
<th>A complex number is any number that can be written in the form $a + bi$, where $a$ and $b$ are real numbers and $i$ is the imaginary unit ($i^2 = -1$). $a$ is called the real part, and $b$ is called the imaginary part.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition and Subtraction of Complex Numbers</td>
<td>Combine like terms. $(a + bi) + (c + di) = (a + c) + (b + d)i$ $(a + bi) - (c + di) = (a - c) + (b - d)i$</td>
</tr>
<tr>
<td>Multiplication of Complex Numbers</td>
<td>Use the definition of $i^2$ and the FOIL method: $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$</td>
</tr>
<tr>
<td>Complex Conjugate</td>
<td>$a + bi$ and $a - bi$ are complex conjugates. The product of complex conjugates is always a real number.</td>
</tr>
</tbody>
</table>

To divide by a complex number, first multiply the dividend and divisor by the **complex conjugate** of the divisor.

**Example 1** Simplify $(6 + i) + (4 - 5i)$.

$$(6 + i) + (4 - 5i) = (6 + 4) + (1 - 5)i = 10 - 4i$$

**Example 3** Simplify $(2 - 5i) \cdot (-4 + 2i)$.

$$(2 - 5i) \cdot (-4 + 2i) = 2(-4) + 2(2i) + (-5i)(-4) + (-5i)(2i) = -8 + 4i + 20i - 10i^2 = -8 + 24i - 10(-1) = 2 + 24i$$

**Example 2** Simplify $(8 + 3i) - (6 - 2i)$.

$$(8 + 3i) - (6 - 2i) = (8 - 6) + [3 - (-2)]i = 2 + 5i$$

**Example 4** Simplify $\frac{3 - i}{2 + 3i}$.

$$\frac{3 - i}{2 + 3i} = \frac{3 - i \cdot 2 - 3i}{2 + 3i} = \frac{6 - 9i - 2i + 3i^2}{4 - 9i^2} = \frac{6 - 11i}{13} = \frac{3 - 11i}{13}$$

**Exercises**

Simplify.

1. $(−4 + 2i) + (6 - 3i)$
2. $(5 - i) - (3 - 2i)$
3. $(6 - 3i) + (4 - 2i)$
4. $(−11 + 4i) - (1 - 5i)$
5. $(8 + 4i) + (8 - 4i)$
6. $(5 + 2i) - (−6 - 3i)$
7. $(2 + i)(3 - i)$
8. $(5 - 2i)(4 - i)$
9. $(4 - 2i)(1 - 2i)$
10. $\frac{5}{3 + i}$
11. $\frac{7 - 13i}{2i}$
12. $\frac{6 - 5i}{3i}$
4-4 Skills Practice

Complex Numbers

Simplify.

1. \( \sqrt{99} \)

2. \( \sqrt{\frac{27}{49}} \)

3. \( \sqrt{52x^3y^5} \)

4. \( \sqrt{-108x^7} \)

5. \( \sqrt{-81x^6} \)

6. \( \sqrt{-23} \cdot \sqrt{-46} \)

7. \( (3i)(-2i)(5i) \)

8. \( i^{11} \)

9. \( i^{65} \)

10. \( (7 - 8i) + (-12 - 4i) \)

11. \( (-3 + 5i) + (18 - 7i) \)

12. \( (10 - 4i) - (7 + 3i) \)

13. \( (7 - 6i)(2 - 3i) \)

14. \( (3 + 4i)(3 - 4i) \)

15. \( \frac{8 - 6i}{3i} \)

16. \( \frac{3i}{4 + 2i} \)

Solve each equation.

17. \( 3x^2 + 3 = 0 \)

18. \( 5x^2 + 125 = 0 \)

19. \( 4x^2 + 20 = 0 \)

20. \( -x^2 - 16 = 0 \)

21. \( x^2 + 18 = 0 \)

22. \( 8x^2 + 96 = 0 \)

Find the values of \( \ell \) and \( m \) that make each equation true.

23. \( 20 - 12i = 5\ell + (4m)i \)

24. \( \ell - 16i = 3 - (2m)i \)

25. \( (4 + \ell) + (2m)i = 9 + 14i \)

26. \( (3 - m) + (7\ell - 14)i = 1 + 7i \)
Simplify.

1. $\sqrt{-36}$
2. $\sqrt{-8} \cdot \sqrt{-32}$
3. $\sqrt{-15} \cdot \sqrt{-25}$
4. $(-3i)(4i)(-5i)$
5. $(7i)^2(6i)$
6. $i^{42}$
7. $i^{55}$
8. $i^{89}$
9. $(5 - 2i) + (-13 - 8i)$
10. $(7 - 6i) + (9 + 11i)$
11. $(-12 + 48i) + (15 + 21i)$
12. $(10 + 15i) - (48 - 30i)$
13. $(28 - 4i) - (10 - 30i)$
14. $(6 - 4i)(6 + 4i)$
15. $(8 - 11i)(8 - 11i)$
16. $(4 + 3i)(2 - 5i)$
17. $(7 + 2i)(9 - 6i)$
18. $\frac{6 + 5i}{-2i}$
19. $\frac{2}{7 - 8i}$
20. $\frac{3 - i}{2 - i}$
21. $\frac{2 - 4i}{1 + 3i}$

Solve each equation.

22. $5m^2 + 35 = 0$
23. $2m^2 + 10 = 0$
24. $4m^2 + 76 = 0$
25. $-2m^2 - 6 = 0$
26. $-5m^2 - 65 = 0$
27. $\frac{3}{4}x^2 + 12 = 0$

Find the values of $\ell$ and $m$ that make each equation true.

28. $15 - 28i = 3\ell + (4m)i$
29. $(6 - \ell) + (3m)i = -12 + 27i$
30. $(3\ell + 4) + (3 - m)i = 16 - 3i$
31. $(7 + m) + (4\ell - 10)i = 3 - 6i$

32. **ELECTRICITY** The impedance in one part of a series circuit is $1 + 3j$ ohms and the impedance in another part of the circuit is $7 - 5j$ ohms. Add these complex numbers to find the total impedance in the circuit.

33. **ELECTRICITY** Using the formula $E = IZ$, find the voltage $E$ in a circuit when the current $I$ is $3 - j$ amps and the impedance $Z$ is $3 + 2j$ ohms.
4-4 Word Problem Practice

Complex Numbers

1. SIGN ERRORS Jennifer and Jessica come up with different answers to the same problem. They had to multiply $(4 + i)(4 - i)$ and give their answer as a complex number. Jennifer claims that the answer is 15 and Jessica claims that the answer is 17. Who is correct? Explain.

2. COMPLEX CONJUGATES You have seen that the product of complex conjugates is always a real number. Show that the sum of complex conjugates is also always a real number.

3. PYTHAGOREAN TRIPLES If three integers $a$, $b$, and $c$ satisfy $a^2 + b^2 = c^2$, then they are called a Pythagorean triple. Suppose that $a$, $b$, and $c$ are a Pythagorean triple. Show that the real and imaginary parts of $(a + bi)^2$, together with the number $c^2$, form another Pythagorean triple.

4. ROTATIONS Complex numbers can be used to perform rotations in the plane. For example, if $(x, y)$ are the coordinates of a point in the plane, then the real and imaginary parts of $i(x + yi)$ are the horizontal and vertical coordinates of the $90^\circ$ counterclockwise rotation of $(x, y)$ about the origin. What are the real and imaginary parts of $i(x + yi)$?

5. ELECTRICAL ENGINEERING Alternating current (AC) in an electrical circuit can be described by complex numbers. In any electrical circuit, $Z$, the impedance in the circuit, is related to the voltage $V$ and the current $I$ by the formula $Z = \frac{V}{I}$. The standard electrical voltage in Europe is 220 volts, so in these problems use $V = 220$.

   a. Find the impedance in a standard European circuit if the current is $22 - 11i$ amps.

   b. Find the current in a standard European circuit if the impedance is $10 - 5i$ watts.

   c. Find the impedance in a standard European circuit if the current is $20i$ amps.
Conjugates and Absolute Value

When studying complex numbers, it is often convenient to represent a complex number by a single variable. For example, we might let \( z = x + yi \). We denote the conjugate of \( z \) by \( \bar{z} \). Thus, \( \bar{z} = x - yi \).

We can define the absolute value of a complex number as follows.

\[
|z| = |x + yi| = \sqrt{x^2 + y^2}
\]

There are many important relationships involving conjugates and absolute values of complex numbers.

**Example 1**
Show \( |z|^2 = z\bar{z} \) for any complex number \( z \).

Let \( z = x + yi \). Then,
\[
z\bar{z} = (x + yi)(x - yi)
= x^2 + y^2
= \sqrt{(x^2 + y^2)^2}
= |z|^2
\]

**Example 2**
Show \( \frac{\bar{z}}{|z|^2} \) is the multiplicative inverse for any nonzero complex number \( z \).

We know \( |z|^2 = z\bar{z} \). If \( z \neq 0 \), then we have \( z\left(\frac{\bar{z}}{|z|^2}\right) = 1 \).

Thus, \( \frac{\bar{z}}{|z|^2} \) is the multiplicative inverse of \( z \).

**Exercises**

For each of the following complex numbers, find the absolute value and multiplicative inverse.

1. \( 2i \)
2. \(-4 - 3i \)
3. \( 12 - 5i \)
4. \( 5 - 12i \)
5. \( 1 + i \)
6. \( \sqrt{3} - i \)
7. \( \frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{3}i \)
8. \( \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \)
9. \( \frac{1}{2} - \frac{\sqrt{3}}{2}i \)
4-5 Study Guide and Intervention

Completing the Square

Square Root Property Use the Square Root Property to solve a quadratic equation that is in the form “perfect square trinomial = constant.”

Example Solve each equation by using the Square Root Property. Round to the nearest hundredth if necessary.

a. \(x^2 - 8x + 16 = 25\)
\(x^2 - 8x + 16 = 25\)
\((x - 4)^2 = 25\)
\(x - 4 = \sqrt{25} \quad \text{or} \quad x - 4 = -\sqrt{25}\)
\(x = 5 + 4 = 9 \quad \text{or} \quad x = -5 + 4 = -1\)
The solution set is \(\{9, -1\}\).

b. \(4x^2 - 20x + 25 = 32\)
\(4x^2 - 20x + 25 = 32\)
\((2x - 5)^2 = 32\)
\(2x - 5 = \sqrt{32} \quad \text{or} \quad 2x - 5 = -\sqrt{32}\)
\(2x - 5 = 4\sqrt{2} \quad \text{or} \quad 2x - 5 = -4\sqrt{2}\)
\(x = \frac{5 \pm 4\sqrt{2}}{2}\)
The solution set is \(\left\{\frac{5 \pm 4\sqrt{2}}{2}\right\}\).

Exercises Solve each equation by using the Square Root Property. Round to the nearest hundredth if necessary.

1. \(x^2 - 18x + 81 = 49\)
2. \(x^2 + 20x + 100 = 64\)
3. \(4x^2 + 4x + 1 = 16\)
4. \(36x^2 + 12x + 1 = 18\)
5. \(9x^2 - 12x + 4 = 4\)
6. \(25x^2 + 40x + 16 = 28\)
7. \(4x^2 - 28x + 49 = 64\)
8. \(16x^2 + 24x + 9 = 81\)
9. \(100x^2 - 60x + 9 = 121\)
10. \(25x^2 + 20x + 4 = 75\)
11. \(36x^2 + 48x + 16 = 12\)
12. \(25x^2 - 30x + 9 = 96\)
4-5 Study Guide and Intervention (continued)  
Completing the Square

Complete the Square  To complete the square for a quadratic expression of the form \(x^2 + bx\), follow these steps.

1. Find \(\frac{b}{2}\)  \(\rightarrow\)  2. Square \(\frac{b}{2}\)  \(\rightarrow\)  3. Add \(\left(\frac{b}{2}\right)^2\) to \(x^2 + bx\).

**Example 1**  Find the value of \(c\) that makes \(x^2 + 22x + c\) a perfect square trinomial. Then write the trinomial as the square of a binomial.

- **Step 1**  \(b = 22; \frac{b}{2} = 11\)
- **Step 2**  \(11^2 = 121\)
- **Step 3**  \(c = 121\)

The trinomial is \(x^2 + 22x + 121\), which can be written as \((x + 11)^2\).

**Example 2**  Solve \(2x^2 - 8x - 24 = 0\) by completing the square.

\[
\begin{align*}
2x^2 - 8x - 24 &= 0 & & \text{Original equation} \\
\frac{2x^2 - 8x - 24}{2} &= \frac{0}{2} & & \text{Divide each side by 2.} \\
x^2 - 4x - 12 &= 0 & & x^2 - 4x - 12 \text{ is not a perfect square.} \\
x^2 - 4x &= 12 & & \text{Add 12 to each side.} \\
x^2 - 4x + 4 = 12 + 4 & & \text{Since } \left(\frac{4}{2}\right)^2 = 4, \text{ add 4 to each side.} \\
(x - 2)^2 &= 16 & & \text{Factor the square.} \\
x - 2 &= \pm 4 & & \text{Square Root Property} \\
x &= 6 \text{ or } x = -2 & & \text{Solve each equation.} \\
\text{The solution set is } \{6, -2\}.
\end{align*}
\]

**Exercises**

Find the value of \(c\) that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

1. \(x^2 - 10x + c\)  
2. \(x^2 + 60x + c\)  
3. \(x^2 - 3x + c\)  

4. \(x^2 + 3.2x + c\)  
5. \(x^2 + \frac{1}{2}x + c\)  
6. \(x^2 - 2.5x + c\)  

Solve each equation by completing the square.

7. \(y^2 - 4y - 5 = 0\)  
8. \(x^2 - 8x - 65 = 0\)  
9. \(w^2 - 10w + 21 = 0\)  

10. \(2x^2 - 3x + 1 = 0\)  
11. \(2x^2 - 13x - 7 = 0\)  
12. \(25x^2 + 40x - 9 = 0\)  

13. \(x^2 + 4x + 1 = 0\)  
14. \(y^2 + 12y + 4 = 0\)  
15. \(t^2 + 3t - 8 = 0\)
4-5 Skills Practice

Completing the Square

Solve each equation by using the Square Root Property. Round to the nearest hundredth if necessary.

1. \(x^2 - 8x + 16 = 1\)  
2. \(x^2 + 4x + 4 = 1\)

3. \(x^2 + 12x + 36 = 25\)  
4. \(4x^2 - 4x + 1 = 9\)

5. \(x^2 + 4x + 4 = 2\)  
6. \(x^2 - 2x + 1 = 5\)

7. \(x^2 - 6x + 9 = 7\)  
8. \(x^2 + 16x + 64 = 15\)

Find the value of \(c\) that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

9. \(x^2 + 10x + c\)  
10. \(x^2 - 14x + c\)

11. \(x^2 + 24x + c\)  
12. \(x^2 + 5x + c\)

13. \(x^2 - 9x + c\)  
14. \(x^2 - x + c\)

Solve each equation by completing the square.

15. \(x^2 - 13x + 36 = 0\)  
16. \(x^2 + 3x = 0\)

17. \(x^2 + x - 6 = 0\)  
18. \(x^2 - 4x - 13 = 0\)

19. \(2x^2 + 7x - 4 = 0\)  
20. \(3x^2 + 2x - 1 = 0\)

21. \(x^2 + 3x - 6 = 0\)  
22. \(x^2 - x - 3 = 0\)

23. \(x^2 = -11\)  
24. \(x^2 - 2x + 4 = 0\)
4-5 Practice

Completing the Square

Solve each equation by using the Square Root Property. Round to the nearest hundredth if necessary.

1. \(x^2 + 8x + 16 = 1\)  
2. \(x^2 + 6x + 9 = 1\)  
3. \(x^2 + 10x + 25 = 16\)

4. \(x^2 - 14x + 49 = 9\)  
5. \(4x^2 + 12x + 9 = 4\)  
6. \(x^2 - 8x + 16 = 8\)

7. \(x^2 - 6x + 9 = 5\)  
8. \(x^2 - 2x + 1 = 2\)  
9. \(9x^2 - 6x + 1 = 2\)

Find the value of \(c\) that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

10. \(x^2 + 12x + c\)  
11. \(x^2 - 20x + c\)  
12. \(x^2 + 11x + c\)

13. \(x^2 + 0.8x + c\)  
14. \(x^2 - 2.2x + c\)  
15. \(x^2 - 0.36x + c\)

16. \(x^2 + \frac{5}{6}x + c\)  
17. \(x^2 - \frac{1}{4}x + c\)  
18. \(x^2 - \frac{5}{3}x + c\)

Solve each equation by completing the square.

19. \(x^2 + 6x + 8 = 0\)  
20. \(3x^2 + x - 2 = 0\)  
21. \(3x^2 - 5x + 2 = 0\)

22. \(x^2 + 18 = 9x\)  
23. \(x^2 - 14x + 19 = 0\)  
24. \(x^2 + 16x - 7 = 0\)

25. \(2x^2 + 8x - 3 = 0\)  
26. \(x^2 + x - 5 = 0\)  
27. \(2x^2 - 10x + 5 = 0\)

28. \(x^2 + 3x + 6 = 0\)  
29. \(2x^2 + 5x + 6 = 0\)  
30. \(7x^2 + 6x + 2 = 0\)

31. GEOMETRY When the dimensions of a cube are reduced by 4 inches on each side, the surface area of the new cube is 864 square inches. What were the dimensions of the original cube?

32. INVESTMENTS The amount of money \(A\) in an account in which \(P\) dollars are invested for 2 years is given by the formula \(A = P(1 + r)^2\), where \(r\) is the interest rate compounded annually. If an investment of $800 in the account grows to $882 in two years, at what interest rate was it invested?
4-5 Word Problem Practice

Completing the Square

1. COMPLETING THE SQUARE
   Samantha needs to solve the equation
   \[ x^2 - 12x = 40. \]
   What must she do to each side of the equation to complete the square?

2. ART
   The area in square inches of the drawing *Foliage* by Paul Cézanne is approximated by the equation
   \[ y = x^2 - 40x + 396. \]
   Complete the square and find the two roots, which are equal to the approximate length and width of the drawing.

3. COMPOUND INTEREST
   Nikki invested $1000 in a savings account with interest compounded annually. After two years the balance in the account is $1210. Use the compound interest formula
   \[ A = P(1 + r)^t \]
   to find the annual interest rate.

4. REACTION TIME
   Lauren was eating lunch when she saw her friend Jason approach. The room was crowded and Jason had to lift his tray to avoid obstacles. Suddenly, a glass on Jason’s lunch tray tipped and fell off the tray. Lauren lunged forward and managed to catch the glass just before it hit the ground. The height \( h \), in feet, of the glass \( t \) seconds after it was dropped is given by
   \[ h = -16t^2 + 4.5. \]
   Lauren caught the glass when it was six inches off the ground. How long was the glass in the air before Lauren caught it?

5. PARABOLAS
   A parabola is modeled by
   \[ y = x^2 - 10x + 28. \]
   Jane’s homework problem requires that she find the vertex of the parabola. She uses the completing square method to express the function in the form
   \[ y = (x - h)^2 + k, \]
   where \((h, k)\) is the vertex of the parabola. Write the function in the form used by Jane.

6. AUDITORIUM SEATING
   The seats in an auditorium are arranged in a square grid pattern. There are 45 rows and 45 columns of chairs. For a special concert, organizers decide to increase seating by adding \( n \) rows and \( n \) columns to make a square pattern of seating \( 45 + n \) seats on a side.
   a. How many seats are there after the expansion?
   b. What is \( n \) if organizers wish to add 1000 seats?
   c. If organizers do add 1000 seats, what is the seating capacity of the auditorium?
The Golden Quadratic Equations

A golden rectangle has the property that its length can be written as \( a + b \), where \( a \) is the width of the rectangle and \( \frac{a + b}{a} = \frac{a}{b} \). Any golden rectangle can be divided into a square and a smaller golden rectangle, as shown.

The proportion used to define golden rectangles can be used to derive two quadratic equations. These are sometimes called golden quadratic equations.

Solve each problem.

1. In the proportion for the golden rectangle, let \( a \) equal 1. Write the resulting quadratic equation and solve for \( b \).

2. In the proportion, let \( b \) equal 1. Write the resulting quadratic equation and solve for \( a \).

3. Describe the difference between the two golden quadratic equations you found in exercises 1 and 2.

4. Show that the positive solutions of the two equations in exercises 1 and 2 are reciprocals.

5. Use the Pythagorean Theorem to find a radical expression for the diagonal of a golden rectangle when \( a = 1 \).

6. Find a radical expression for the diagonal of a golden rectangle when \( b = 1 \).
4-6 Study Guide and Intervention

The Quadratic Formula and the Discriminant

Quadratic Formula The Quadratic Formula can be used to solve any quadratic equation once it is written in the form $ax^2 + bx + c = 0$.

| Quadratic Formula | The solutions of $ax^2 + bx + c = 0$, with $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. |

Example Solve $x^2 - 5x = 14$ by using the Quadratic Formula.

Rewrite the equation as $x^2 - 5x - 14 = 0$.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Quadratic Formula

$= \frac{-(5) \pm \sqrt{(-5)^2 - 4(1)(-14)}}{2(1)}$

Replace $a$ with 1, $b$ with $-5$, and $c$ with $-14$.

Simplify.

$= \frac{5 \pm \sqrt{81}}{2}$

$= \frac{5 \pm 9}{2}$

$= \frac{5 + 9}{2}$ or $\frac{5 - 9}{2}$

$= \frac{7}{2} \text{ or } -2$

The solutions are $-2$ and $7$.

Exercises Solve each equation by using the Quadratic Formula.

1. $x^2 + 2x - 35 = 0$
2. $x^2 + 10x + 24 = 0$
3. $x^2 - 11x + 24 = 0$

4. $4x^2 + 19x - 5 = 0$
5. $14x^2 + 9x + 1 = 0$
6. $2x^2 - x - 15 = 0$

7. $3x^2 + 5x = 2$
8. $2y^2 + y - 15 = 0$
9. $3x^2 - 16x + 16 = 0$

10. $8x^2 + 6x - 9 = 0$
11. $r^2 - \frac{3r}{5} + \frac{2}{25} = 0$
12. $x^2 - 10x - 50 = 0$

13. $x^2 + 6x - 23 = 0$
14. $4x^2 - 12x - 63 = 0$
15. $x^2 - 6x + 21 = 0$
The expression under the radical sign, $b^2 - 4ac$, in the Quadratic Formula is called the **discriminant**.

<table>
<thead>
<tr>
<th>Discriminant</th>
<th>Type and Number of Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b^2 - 4ac &gt; 0$ and a perfect square</td>
<td>2 rational roots</td>
</tr>
<tr>
<td>$b^2 - 4ac &gt; 0$, but <strong>not</strong> a perfect square</td>
<td>2 irrational roots</td>
</tr>
<tr>
<td>$b^2 - 4ac = 0$</td>
<td>1 rational root</td>
</tr>
<tr>
<td>$b^2 - 4ac &lt; 0$</td>
<td>2 complex roots</td>
</tr>
</tbody>
</table>

**Example** Find the value of the discriminant for each equation. Then describe the number and type of roots for the equation.

a. $2x^2 + 5x + 3$
   The discriminant is $b^2 - 4ac = 5^2 - 4(2)(3) = 1$. The discriminant is a perfect square, so the equation has 2 rational roots.

b. $3x^2 - 2x + 5$
   The discriminant is $b^2 - 4ac = (-2)^2 - 4(3)(5) = -56$. The discriminant is negative, so the equation has 2 complex roots.

**Exercises**

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.
b. Describe the number and type of roots.
c. Find the exact solutions by using the Quadratic Formula.

1. $p^2 + 12p = -4$
2. $9x^2 - 6x + 1 = 0$
3. $2x^2 - 7x - 4 = 0$
4. $x^2 + 4x - 4 = 0$
5. $5x^2 - 36x + 7 = 0$
6. $4x^2 - 4x + 11 = 0$
7. $x^2 - 7x + 6 = 0$
8. $m^2 - 8m = -14$
9. $25x^2 - 40x = -16$
10. $4x^2 + 20x + 29 = 0$
11. $6x^2 + 26x + 8 = 0$
12. $4x^2 - 4x - 11 = 0$
Skills Practice

The Quadratic Formula and the Discriminant

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.
b. Describe the number and type of roots.
c. Find the exact solutions by using the Quadratic Formula.

1. \( x^2 - 8x + 16 = 0 \)  
2. \( x^2 - 11x - 26 = 0 \)

3. \( 3x^2 - 2x = 0 \)  
4. \( 20x^2 + 7x - 3 = 0 \)

5. \( 5x^2 - 6 = 0 \)  
6. \( x^2 - 6 = 0 \)

7. \( x^2 + 8x + 13 = 0 \)  
8. \( 5x^2 - x - 1 = 0 \)

9. \( x^2 - 2x - 17 = 0 \)  
10. \( x^2 + 49 = 0 \)

11. \( x^2 - x + 1 = 0 \)  
12. \( 2x^2 - 3x = -2 \)

Solve each equation by using the Quadratic Formula.

13. \( x^2 = 64 \)  
14. \( x^2 - 30 = 0 \)

15. \( x^2 - x = 30 \)  
16. \( 16x^2 - 24x - 27 = 0 \)

17. \( x^2 - 4x - 11 = 0 \)  
18. \( x^2 - 8x - 17 = 0 \)

19. \( x^2 + 25 = 0 \)  
20. \( 3x^2 + 36 = 0 \)

21. \( 2x^2 + 10x + 11 = 0 \)  
22. \( 2x^2 - 7x + 4 = 0 \)

23. \( 8x^2 + 1 = 4x \)  
24. \( 2x^2 + 2x + 3 = 0 \)

25. PARACHUTING  Ignoring wind resistance, the distance \( d(t) \) in feet that a parachutist falls in \( t \) seconds can be estimated using the formula \( d(t) = 16t^2 \). If a parachutist jumps from an airplane and falls for 1100 feet before opening her parachute, how many seconds pass before she opens the parachute?
The Quadratic Formula and the Discriminant

Solve each equation by using the Quadratic Formula.

1. \(7x^2 - 5x = 0\)  
2. \(4x^2 - 9 = 0\)

3. \(3x^2 + 8x = 3\)  
4. \(x^2 - 21 = 4x\)

5. \(3x^2 - 13x + 4 = 0\)  
6. \(15x^2 + 22x = -8\)

7. \(x^2 - 6x + 3 = 0\)  
8. \(x^2 - 14x + 53 = 0\)

9. \(3x^2 = -54\)  
10. \(25x^2 - 20x - 6 = 0\)

11. \(4x^2 - 4x + 17 = 0\)  
12. \(8x - 1 = 4x^2\)

13. \(x^2 = 4x - 15\)  
14. \(4x^2 - 12x + 7 = 0\)

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

15. \(x^2 - 16x + 64 = 0\)

16. \(x^2 = 3x\)

17. \(9x^2 - 24x + 16 = 0\)

18. \(x^2 - 3x = 40\)

19. \(3x^2 + 9x - 2 = 0\)

20. \(2x^2 + 7x = 0\)

21. \(5x^2 - 2x + 4 = 0\)

22. \(12x^2 - x - 6 = 0\)

23. \(7x^2 + 6x + 2 = 0\)

24. \(12x^2 + 2x - 4 = 0\)

25. \(6x^2 - 2x - 1 = 0\)

26. \(x^2 + 3x + 6 = 0\)

27. \(4x^2 - 3x^2 - 6 = 0\)

28. \(16x^2 - 8x + 1 = 0\)

29. \(2x^2 - 5x - 6 = 0\)

30. **GRAVITATION** The height \(h(t)\) in feet of an object \(t\) seconds after it is propelled straight up from the ground with an initial velocity of 60 feet per second is modeled by the equation \(h(t) = -16t^2 + 60t\). At what times will the object be at a height of 56 feet?

31. **STOPPING DISTANCE** The formula \(d = 0.05s^2 + 1.1s\) estimates the minimum stopping distance \(d\) in feet for a car traveling \(s\) miles per hour. If a car stops in 200 feet, what is the fastest it could have been traveling when the driver applied the brakes?
1. **PARABOLAS** The graph of a quadratic equation of the form \( y = ax^2 + bx + c \) is shown below.

![Graph of a quadratic equation](image)

Is the discriminant \( b^2 - 4ac \) positive, negative, or zero? Explain.

2. **TANGENT** Kathleen is trying to find \( b \) so that the \( x \)-axis is tangent to the parabola \( y = x^2 + bx + 4 \). She finds one value that works, \( b = 4 \). Is this the only value that works? Explain.

3. **SPORTS** In 1990, American Randy Barnes set the world record for the shot put. His throw can be described by the equation \( y = -16x^2 + 368x \). Use the Quadratic Formula to find how far his throw was to the nearest foot.

4. **EXAMPLES** Give an example of a quadratic function \( f(x) \) that has the following properties.
   
   I. The discriminant of \( f \) is zero.
   
   II. There is no real solution of the equation \( f(x) = 10 \).

   Sketch the graph of \( x = f(x) \).

5. **TANGENTS** The graph of \( y = x^2 \) is a parabola that passes through the point at \((1, 1)\). The line \( y = mx - m + 1 \), where \( m \) is a constant, also passes through the point at \((1, 1)\).

   a. To find the points of intersection between the line \( y = mx - m + 1 \) and the parabola \( y = x^2 \), set \( x^2 = mx - m + 1 \) and then solve for \( x \). Rearranging terms, this equation becomes \( x^2 - mx + m - 1 = 0 \). What is the discriminant of this equation?

   b. For what value of \( m \) is there only one point of intersection? Explain the meaning of this in terms of the corresponding line and the parabola.
Sum and Product of Roots

Sometimes you may know the roots of a quadratic equation without knowing the equation itself. Using your knowledge of factoring to solve an equation, you can work backward to find the quadratic equation. The rule for finding the sum and product of roots is as follows:

If the roots of \( ax^2 + bx + c = 0 \), with \( a \neq 0 \), are \( s_1 \) and \( s_2 \), then \( s_1 + s_2 = -\frac{b}{a} \) and \( s_1 \cdot s_2 = \frac{c}{a} \).

**Example**

Write a quadratic equation that has the roots 3 and \(-8\).

The roots are \( x = 3 \) and \( x = -8 \).

\[
3 + (-8) = -5 \quad \text{Add the roots.}
\]

\[
3(-8) = -24 \quad \text{Multiply the roots.}
\]

Equation: \( x^2 + 5x - 24 = 0 \)

**Exercises**

Write a quadratic equation that has the given roots.

1. 6, \(-9\)  
   2. 5, \(-1\)  
   3. 6, 6  
   
4. \(4 \pm \sqrt{3}\)  
   5. \(-\frac{2}{5}, \frac{2}{7}\)  
   6. \(-2 \pm 3 \sqrt{5}\) \(\frac{7}{7}\)

Find \( k \) such that the number given is a root of the equation.

7. 7; \(2x^2 + kx - 21 = 0\)  
   8. \(-2; x^2 - 13x + k = 0\)
### Spreadsheet Activity

**Approximating the Real Zeros of Polynomials**

You have learned the Location Principle, which can be used to approximate the real zeros of a polynomial.

In the spreadsheet above, the positive real zero of \( f(x) = x^2 - 2 \) can be approximated in the following way. Set the spreadsheet preference to manual calculation. The values in A2 and B2 are the endpoints of a range of values. The values in D2 through J2 are values equally in the interval from A2 to B2. The formulas for these values are A2, A2 + (B2 - A2)/6, A2 + 2*(B2 - A2)/6, A2 + 3*(B2 - A2)/6, A2 + 4*(B2 - A2)/6, A2 + 5*(B2 - A2)/6, and B2, respectively.

Row 3 gives the function values at these points. The function \( f(x) = x^2 - 2 \) is entered into the spreadsheet in Cell D3 as D2^2 - 2. This function is then copied to the remaining cells in the row.

You can use this spreadsheet to study the function values at the points in cells D2 through J2. The value in cell F3 is positive and the value in cell G3 is negative, so there must be a zero between -1.6667 and 0. Enter these values in cells A2 and B2, respectively, and recalculate the spreadsheet. (You will have to recalculate a number of times.) The result is a new table from which you can see that there is a zero between 1.41414 and 1.414306. Because these values agree to three decimal places, the zero is about 1.414. This can be verified by using algebra.

By solving \( x^2 - 2 = 0 \), we obtain \( x = \pm \sqrt{2} \). The positive root is \( x = \pm \sqrt{2} = 1.414213 \ldots \), which verifies the result.

### Exercises

1. Use a spreadsheet like the one above to approximate the zero of \( f(x) = 3x - 2 \) to three decimal places. Then verify your answer by using algebra to find the exact value of the root.

2. Use a spreadsheet like the one above to approximate the real zeros of \( f(x) = x^2 + 2x + 0.5 \). Round your answer to four decimal places. Then, verify your answer by using the quadratic formula.

3. Use a spreadsheet like the one above to approximate the real zero of \( f(x) = x^3 - \frac{3}{2}x^2 - 6x - 2 \) between -0.4 and -0.3.
4-7 Study Guide and Intervention

Transformations of Quadratic Graphs

Write Quadratic Equations in Vertex Form A quadratic function is easier to graph when it is in vertex form. You can write a quadratic function of the form \( y = ax^2 + bx + c \) in vertex form by completing the square.

Example Write \( y = 2x^2 - 12x + 25 \) in vertex form. Then graph the function.

\[
\begin{align*}
  y &= 2x^2 - 12x + 25 \\
  y &= 2(x^2 - 6x) + 25 \\
  y &= 2(x^2 - 6x + 9) + 25 - 18 \\
  y &= 2(x - 3)^2 + 7 \\
  \text{The vertex form of the equation is } y &= 2(x - 3)^2 + 7.
\end{align*}
\]

Exercises

Write each equation in vertex form. Then graph the function.

1. \( y = x^2 - 10x + 32 \) 
2. \( y = x^2 + 6x \) 
3. \( y = x^2 - 8x + 6 \) 

4. \( y = -4x^2 + 16x - 11 \) 
5. \( y = 3x^2 - 12x + 5 \) 
6. \( y = 5x^2 - 10x + 9 \)
Transformations of Quadratic Graphs

Parabolas can be transformed by changing the values of the constants $a$, $h$, and $k$ in the vertex form of a quadratic equation:

$$y = a(x - h)^2 + k.$$

- The sign of $a$ determines whether the graph opens upward ($a > 0$) or downward ($a < 0$).
- The absolute value of $a$ also causes a dilation (enlargement or reduction) of the parabola. The parabola becomes narrower if $|a| > 1$ and wider if $|a| < 1$.
- The value of $h$ translates the parabola horizontally. Positive values of $h$ slide the graph to the right and negative values slide the graph to the left.
- The value of $k$ translates the graph vertically. Positive values of $k$ slide the graph upward and negative values slide the graph downward.

Example

Graph $y = (x + 7)^2 + 3$.

- Rewrite the equation as $y = [x - (-7)]^2 + 3$.
- Because $h = -7$ and $k = 3$, the vertex is at $(-7, 3)$. The axis of symmetry is $x = -7$. Because $a = 1$, we know that the graph opens up, and the graph is the same width as the graph of $y = x^2$.
- Translate the graph of $y = x^2$ seven units to the left and three units up.

Exercises

Graph each function.

1. $y = -2x^2 + 2$
2. $y = -3(x - 1)^2$
3. $y = 2(x + 2)^2 + 3$
4-7 Skills Practice

Transformations of Quadratic Graphs

Write each quadratic function in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.

1. \( y = (x - 2)^2 \)  
2. \( y = -x^2 + 4 \)  
3. \( y = x^2 - 6 \)

4. \( y = -3(x + 5)^2 \)  
5. \( y = -5x^2 + 9 \)  
6. \( y = (x - 2)^2 - 18 \)

7. \( y = x^2 - 2x - 5 \)  
8. \( y = x^2 + 6x + 2 \)  
9. \( y = -3x^2 + 24x \)

Graph each function.

10. \( y = (x - 3)^2 - 1 \)  
11. \( y = (x + 1)^2 + 2 \)  
12. \( y = -(x - 4)^2 - 4 \)

13. \( y = -\frac{1}{2}(x + 2)^2 \)  
14. \( y = -3x^2 + 4 \)  
15. \( y = x^2 + 6x + 4 \)
4-7 Practice

Transformations of Quadratic Graphs

Write each equation in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.

1. \( y = -6x^2 - 24x - 25 \)  
2. \( y = 2x^2 + 2 \)  
3. \( y = -4x^2 + 8x \)

4. \( y = x^2 + 10x + 20 \)  
5. \( y = 2x^2 + 12x + 18 \)  
6. \( y = 3x^2 - 6x + 5 \)

7. \( y = -2x^2 - 16x - 32 \)  
8. \( y = -3x^2 + 18x - 21 \)  
9. \( y = 2x^2 + 16x + 29 \)

Graph each function.

10. \( y = (x + 3)^2 - 1 \)  
11. \( y = -x^2 + 6x - 5 \)  
12. \( y = 2x^2 - 2x + 1 \)

13. Write an equation for a parabola with vertex at \((1, 3)\) that passes through \((-2, -15)\).

14. Write an equation for a parabola with vertex at \((-3, 0)\) that passes through \((3, 18)\).

15. BASEBALL The height \( h \) of a baseball \( t \) seconds after being hit is given by \( h(t) = -16t^2 + 80t + 3 \). What is the maximum height that the baseball reaches, and when does this occur?

16. SCULPTURE A modern sculpture in a park contains a parabolic arc that starts at the ground and reaches a maximum height of 10 feet after a horizontal distance of 4 feet. Write a quadratic function in vertex form that describes the shape of the outside of the arc, where \( y \) is the height of a point on the arc and \( x \) is its horizontal distance from the left-hand starting point of the arc.
1. **ARCHES** A parabolic arch is used as a bridge support. The graph of the arch is shown below.

If the equation that corresponds to this graph is written in the form \( y + a(x - h)^2 + k \), what are \( h \) and \( k \)?

2. **TRANSLATIONS** For a computer animation, Barbara uses the quadratic function \( f(x) = -42(x - 20)^2 + 16800 \) to help her simulate an object tossed on another planet. For one skit, she had to use the function \( f(x + 5) - 8000 \) instead of \( f(x) \). Where is the vertex of the graph of \( y = f(x + 5) - 8000 \)?

3. **BRIDGES** The shape formed by the main cables of the Golden Gate Bridge approximately follows the equation \( y = 0.0002x^2 - 0.23x + 227 \). Graph the parabola formed by one of the cables.

4. **WATER JETS** The graph shows the path of a jet of water.

The equation corresponding to this graph is \( y = a(x - h)^2 + k \). What are \( a \), \( h \), and \( k \)?

5. **PROFIT** A theater operator predicts that the theater can make \( -4x^2 + 160x \) dollars per show if tickets are priced at \( x \) dollars.

a. Rewrite the equation \( y = -4x^2 + 160x \) in the form \( y = a(x - h)^2 + k \).

b. What is the vertex of the parabola and what is its axis of symmetry?

c. Graph the parabola.
A Shortcut to Complex Roots

When graphing a quadratic function, the real roots are shown in the graph. You have learned that quadratic functions can also have imaginary roots that cannot be seen on the graph of the function. However, there is a way to graphically represent the complex roots of a quadratic function.

Example

Find the complex roots of the quadratic function \( y = x^2 - 4x + 5 \).

Step 1 Graph the function.

![Graph of the quadratic function](image)

Step 2 Reflect the graph over the horizontal line containing the vertex. In this example, the vertex is (2, 1).

![Reflected graph](image)

Step 3 The real part of the complex root is the point halfway between the \( x \)-intercepts of the reflected graph and the imaginary part of the complex roots are \( + \) and \( - \) half the distance between the \( x \)-intercepts of the reflected graph. So, in this example, the complex roots are \( 2 + 1i \) and \( 2 - 1i \).

Exercises

Using this method, find the complex roots of the following quadratic functions.

1. \( y = x^2 + 2x + 5 \)
2. \( y = x^2 + 4x + 8 \)
3. \( y = x^2 + 6x + 13 \)
4. \( y = x^2 + 2x + 17 \)
Graph Quadratic Inequalities To graph a quadratic inequality in two variables, use the following steps:

1. Graph the related quadratic equation, \( y = ax^2 + bx + c \).
   Use a dashed line for < or >; use a solid line for \( \leq \) or \( \geq \).

2. Test a point inside the parabola.
   If it satisfies the inequality, shade the region inside the parabola; otherwise, shade the region outside the parabola.

**Example** Graph the inequality \( y > x^2 + 6x + 7 \).

First graph the equation \( y = x^2 + 6x + 7 \). By completing the square, you get the vertex form of the equation \( y = (x + 3)^2 - 2 \), so the vertex is \((-3, -2)\). Make a table of values around \( x = -3 \), and graph. Since the inequality includes >, use a dashed line.

Test the point \((-3, 0)\), which is inside the parabola. Since \((-3)^2 + 6(-3) + 7 = -2\), and \(0 > -2\), \((-3, 0)\) satisfies the inequality. Therefore, shade the region inside the parabola.

**Exercises**

Graph each inequality.

1. \( y > x^2 - 8x + 17 \)
2. \( y \leq x^2 + 6x + 4 \)
3. \( y \geq x^2 + 2x + 2 \)
4. \( y < -x^2 + 4x - 6 \)
5. \( y \geq 2x^2 + 4x \)
6. \( y > -2x^2 - 4x + 2 \)
Solve Quadratic Inequalities  Quadratic inequalities in one variable can be solved graphically or algebraically.

**Graphical Method**

To solve $ax^2 + bx + c < 0$:
First graph $y = ax^2 + bx + c$. The solution consists of the $x$-values for which the graph is below the $x$-axis.

To solve $ax^2 + bx + c > 0$:
First graph $y = ax^2 + bx + c$. The solution consists of the $x$-values for which the graph is above the $x$-axis.

**Algebraic Method**

Find the roots of the related quadratic equation by factoring, completing the square; or using the Quadratic Formula.

2 roots divide the number line into 3 intervals.
Test a value in each interval to see which intervals are solutions.

If the inequality involves $\leq$ or $\geq$, the roots of the related equation are included in the solution set.

**Example** Solve the inequality $x^2 - x - 6 \leq 0$.

First find the roots of the related equation $x^2 - x - 6 = 0$. The equation factors as $(x - 3)(x + 2) = 0$, so the roots are 3 and $-2$. The graph opens up with $x$-intercepts 3 and $-2$, so it must be on or below the $x$-axis for $-2 \leq x \leq 3$. Therefore the solution set is $\{x \mid -2 \leq x \leq 3\}$.

**Exercises**

Solve each inequality.

1. $x^2 + 2x < 0$
2. $x^2 - 16 < 0$
3. $0 < 6x - x^2 - 5$
4. $c^2 \leq 4$
5. $2m^2 - m < 1$
6. $y^2 < -8$
7. $x^2 - 4x - 12 < 0$
8. $x^2 + 9x + 14 > 0$
9. $-x^2 + 7x - 10 \geq 0$
10. $2x^2 + 5x - 3 \leq 0$
11. $4x^2 - 23x + 15 > 0$
12. $-6x^2 - 11x + 2 < 0$
13. $2x^2 - 11x + 12 \geq 0$
14. $x^2 - 4x + 5 < 0$
15. $3x^2 - 16x + 5 < 0$
4-8 Skills Practice

Quadratic Inequalities

Graph each inequality.

1. \( y \geq x^2 - 4x + 4 \)  
2. \( y \leq x^2 - 4 \)  
3. \( y > x^2 + 2x - 5 \)

![Graphs of the inequalities](image)

Solve each inequality by graphing.

4. \( x^2 - 6x + 9 \leq 0 \)  
5. \( -x^2 - 4x + 32 \geq 0 \)  
6. \( x^2 + x - 10 > 10 \)

![Graphs of the inequalities](image)

Solve each inequality algebraically.

7. \( x^2 - 3x - 10 < 0 \)  
8. \( x^2 + 2x - 35 \geq 0 \)

9. \( x^2 - 18x + 81 \leq 0 \)  
10. \( x^2 \leq 36 \)

11. \( x^2 - 7x > 0 \)  
12. \( x^2 + 7x + 6 < 0 \)

13. \( x^2 + x - 12 > 0 \)  
14. \( x^2 + 9x + 18 \leq 0 \)

15. \( x^2 - 10x + 25 \geq 0 \)  
16. \( -x^2 - 2x + 15 \geq 0 \)

17. \( x^2 + 3x > 0 \)  
18. \( 2x^2 + 2x > 4 \)

19. \( -x^2 - 64 \leq -16x \)  
20. \( 9x^2 + 12x + 9 < 0 \)
### 4-8 Practice

**Quadratic Inequalities**

**Graph each inequality.**

1. \( y \leq x^2 + 4 \)
   - [Graph]
2. \( y > x^2 + 6x + 6 \)
   - [Graph]
3. \( y < 2x^2 - 4x - 2 \)
   - [Graph]

**Solve each inequality.**

4. \( x^2 + 2x + 1 > 0 \)
5. \( x^2 - 3x + 2 \leq 0 \)
6. \( x^2 + 10x + 7 \geq 0 \)
7. \( x^2 - x - 20 > 0 \)
8. \( x^2 - 10x + 16 < 0 \)
9. \( x^2 + 4x + 5 \leq 0 \)
10. \( x^2 + 14x + 49 \geq 0 \)
11. \( x^2 - 5x > 14 \)
12. \( -x^2 - 15 \leq 8x \)
13. \( -x^2 + 5x - 7 \leq 0 \)
14. \( 9x^2 + 36x + 36 \leq 0 \)
15. \( 9x \leq 12x^2 \)
16. \( 4x^2 + 4x + 1 > 0 \)
17. \( 5x^2 + 10 \geq 27x \)
18. \( 9x^2 + 31x + 12 \leq 0 \)

**19. FENCING** Vanessa has 180 feet of fencing that she intends to use to build a rectangular play area for her dog. She wants the play area to enclose at least 1800 square feet. What are the possible widths of the play area?

**20. BUSINESS** A bicycle maker sold 300 bicycles last year at a profit of $300 each. The maker wants to increase the profit margin this year, but predicts that each $20 increase in profit will reduce the number of bicycles sold by 10. How many $20 increases in profit can the maker add in and expect to make a total profit of at least $100,000?
1. **HUTS** The space inside a hut is shaded in the graph. The parabola is described by the equation \(y = -\frac{4}{5}(x - 1)^2 + 4\).

Write an inequality that describes the shaded region.

2. **DISCRIMINANTS** Consider the equation \(ax^2 + bx + c = 0\). Assume that the discriminant is zero and that \(a\) is positive. What are the solutions of the inequality \(ax^2 + bx + c \leq 0\)?

3. **KIOSKS** Caleb is designing a kiosk by wrapping a piece of sheet metal with dimensions \(x + 5\) inches by \(4x + 8\) inches into a cylindrical shape. Ignoring cost, Caleb would like a kiosk that has a surface area of at least 4480 square inches. What values of \(x\) satisfy this condition?

4. **DAMS** The Hoover Dam is a concrete arch dam designed to hold the water of Lake Mead. At its center, the dam’s height is approximately 725 feet, and the dam varies from 45 to 660 feet thick. The dark line on this sketch of the cross-section of the dam is a parabola.

   a. Write an equation for the Hoover Dam parabola. Let the height be the \(y\)-value of the parabola and the thickness be the \(x\)-value of the parabola. (Hint: the equation will be in the form: \(y = k(x – \text{maximum thickness}) + \text{maximum height}\).)

   b. Using your equation, graph the parabola of the Hoover Dam for \(45 \leq x \leq 660\).

   c. Estimate to the nearest foot the thickness of the dam when the height is 200 feet.
Graphing Absolute Value Inequalities

You can solve absolute value inequalities by graphing in much the same manner you graphed quadratic inequalities. Graph the related absolute function for each inequality by using a graphing calculator. For > and ≥, identify the x-values, if any, for which the graph lies below the x-axis. For < and ≤, identify the x values, if any, for which the graph lies above the x-axis.

For each inequality, make a sketch of the related graph and find the solutions rounded to the nearest hundredth.

1. \(|x - 3| > 0\\)
2. \(|x| - 6 < 0\\)
3. \(-|x + 4| + 8 < 0\\)

4. \(2|x + 6| - 2 ≥ 0\\)
5. \(3x - 3 | ≥ 0\\)
6. \(|x - 7| < 5\\)

7. \(|7x - 1| > 13\\)
8. \(|x - 3.6| ≤ 4.2\\)
9. \(|2x + 5| ≤ 7\\)
Graphing Calculator Activity

Quadratic Inequalities and the Test Menu

The inequality symbols, called relational operators, in the TEST menu can be used to display the solution of a quadratic inequality. Another method that can be used to find the solution set of a quadratic inequality is to graph each side of an inequality separately. Examine the graphs and use the intersect function to determine the range of values for which the inequality is true.

Example 1  Solve \( x^2 + x \geq 6 \).

Place the calculator in Dot mode. Enter the inequality into \( Y_1 \). Then trace the graph and describe the solution as an inequality.

Keystrokes: \( Y= \) X,T,\( \theta \),n \( x^2 \) + X,T,\( \theta \),n \( 2nd \) [TEST] 4 6 ZOOM 4.

Use TRACE to determine the endpoints of the segments. These values are used to express the solution of the inequality, \( \{ x \mid x \geq -3 \text{ or } x \geq 2 \} \).

Example 2  Solve \( 2x^2 + 4x - 5 \leq 3 \).

Place the left side of the inequality in \( Y_1 \) and the right side in \( Y_2 \). Determine the points of intersection. Use the intersection points to express the solution set of the inequality. Be sure to set the calculator to Connected mode.

Keystrokes: \( Y= \) 2 X,T,\( \theta \),n \( x^2 \) + 4 X,T,\( \theta \),n - 5 ENTER 3 ENTER ZOOM 6.

Press 2nd [CALC] 5 and use the \( \rightarrow \) key to move the cursor to the left of the first intersection point. Press ENTER. Then move the cursor to the right of the intersection point and press ENTER ENTER. One of the values used in the solution set is displayed. Repeat the procedure on the other intersection point.

The solution is \( \{ x \mid -3.24 \leq x \leq 1.24 \} \).

Exercises

Solve each inequality.

1. \(-x^2 - 10x - 21 < 0\)  
2. \(x^2 - 9 < 0\)  
3. \(x^2 + 10x + 25 \leq 0\)

4. \(x^2 + 3x \leq 28\)  
5. \(2x^2 + x \geq 3\)  
6. \(4x^2 + 12x + 9 > 0\)

7. \(23 > -x^2 + 10x\)  
8. \(x^2 - 4x - 13 \leq 0\)  
9. \((x + 1)(x - 3) > 0\)
4 Student Recording Sheet

Use this recording sheet with pages 298–299 of the Student Edition.

Multiple Choice

Read each question. Then fill in the correct answer.

1. ☐ ☐ ☐ ☐
2. ☐ ☐ ☐ ☐
3. ☐ ☐ ☐ ☐
4. ☐ ☐ ☐ ☐
5. ☐ ☐ ☐ ☐
6. ☐ ☐ ☐ ☐
7. ☐ ☐ ☐ ☐
8. ☐ ☐ ☐ ☐

Short Response/Grided Response

Record your answer in the blank.

For gridded response, also enter your answer in the grid by writing each number or symbol in a box. Then fill in the corresponding circle for that number or symbol.

9. ———— (grid in) 9. ☐ ☐ ☐ 10. ———— (grid in)
10. ———— (grid in)

11. a. ————
    b. ————
    c. ————

12. ————
13. ————

Extended Response

Record your answers for Questions 14–16 on the back of this paper.
General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended response questions require the student to show work.

- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is not considered a fully correct response.

- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercises 14–16 Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>Specific Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>For Exercise 14, the three possible results of the discriminant (positive, negative, or zero) are explained. Positive has two real roots, zero has one real root, and negative has no real roots. For Exercise 15, a system of equation is written in part a. One equation should be based on the cost of the printers; the second equation will be based on the number of each that exist. In part b, the system of equations is set up in a matrix and used in part c to find the inverse to solve the system of equations. For Exercise 16, the quadratic equation is graphed in part a. The vertex of this graph will be used in parts b and c to solve for maximum height and time.</td>
</tr>
<tr>
<td>3</td>
<td>A generally correct solution, but may contain minor flaws in reasoning or computation.</td>
</tr>
<tr>
<td>2</td>
<td>A partially correct interpretation and/or solution to the problem.</td>
</tr>
<tr>
<td>1</td>
<td>A correct solution with no evidence or explanation.</td>
</tr>
<tr>
<td>0</td>
<td>An incorrect solution indicating no mathematical understanding of the concept or task, or no solution given.</td>
</tr>
</tbody>
</table>
4 Chapter 4 Quiz 1
(Lessons 4-1 and 4-2)

For Questions 1 and 2, consider \( f(x) = x^2 + 2x - 3 \).

1. Find the \( y \)-intercept, the equation of the axis of symmetry, and the \( x \)-coordinate of the vertex.

2. Graph the function, labeling the \( y \)-intercept, vertex, and axis of symmetry.

3. MULTIPLE CHOICE Determine the maximum or minimum value of \( f(x) = 2x^2 - 8x + 9 \).

   A min. 1       B max. 1       C min. 2       D max. 2

3. ________________

Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. \( x^2 - 2x = 3 \)

5. \( x^2 + 4x - 7 = 0 \)

4. ________________

5. ________________

4 Chapter 4 Quiz 2
(Lessons 4-3 and 4-4)

For Questions 1 and 2, solve each equation by factoring.

1. \( 3x^2 = 10 - 13x \)

2. \( x^2 + 4x = 45 \)

2. ________________

3. MULTIPLE CHOICE Solve \( 5x^2 + 100 = 0 \).

   A \( \pm 2\sqrt{5} \)       B \( \pm 10\sqrt{5} \)       C \( \pm 2i\sqrt{5} \)       D \( \pm 10i\sqrt{5} \)

3. ________________

Write a quadratic equation with the given roots. Write the equation in the form \( ax^2 + bx + c = 0 \), where \( a \), \( b \), and \( c \) are integers.

4. \(-6 \text{ and } 2\)

5. \( \frac{2}{3} \text{ and } -4\)

4. ________________

5. ________________

Simplify.

6. \( \sqrt{-80} \)

7. \( \sqrt{-6} \cdot \sqrt{-12} \)

8. \( (6 - 9i) - (17 - 12i) \)

9. \( (7 - 3i)(8 + 4i) \)

10. \( \frac{2 + i}{3 - i} \)

10. ________________
Solve each equation by using the Square Root Property.

1. \( x^2 + 8x + 16 = 36 \)

2. \( x^2 - 2x + 1 = 45 \)

3. **MULTIPLE CHOICE** What is the solution of \( x^2 - 10x = 11 \)?

   - A \( \{-11, 1\} \)
   - B \( \{-1, 11\} \)
   - C \( -5 \pm \sqrt{14} \)
   - D \( 5 \pm \sqrt{14} \)

4. Solve \( x^2 - 4x = 1 \) by using the Quadratic Formula. Find exact solutions.

5. Find the value of the discriminant for \( 3x^2 = 6x - 11 \). Then describe the number and type of roots for the equation.

---

1. Graph \( y = -(x - 2)^2 - 1 \). Show and label the vertex and axis of symmetry.

2. Write an equation for the parabola whose vertex is at \((-5, 0)\) and passes through \((0, 50)\).

3. Graph \( y \leq -\frac{1}{3}(x + 2)^2 + 3 \).

4. Use the graph of its related function to write the solutions of \(-x^2 + 6x - 5 > 0\).

5. **MULTIPLE CHOICE** What is the solution of \( 4x^2 + 1 \geq 4x \)?

   - A all reals
   - B empty set
   - C \( \left\{ x \mid x \geq \frac{1}{2} \right\} \)
   - D \( \left\{ x \mid x \leq \frac{1}{2} \right\} \)
Part I  Write the letter for the correct answer in the blank at the right of each question.

1. Which function is graphed?
   A  $f(x) = x^2 - 2x - 3$
   B  $f(x) = x^2 + 2x - 3$
   C  $f(x) = x^2 + x - 3$
   D  $f(x) = (x - 3)^2$

2. By the Zero Product Property, if $(2x - 1)(x - 5) = 0$, then ______.
   F  $x = 1$ or $x = 5$
   H  $x = \frac{1}{2}$ or $x = 5$
   G  $x = -\frac{1}{2}$ or $x = -5$
   J  $x = -1$ or $x = -5$

3. Write a quadratic equation with 7 and $\frac{2}{5}$ as its roots. Write the equation in the form $ax^2 + bx + c = 0$, where $a$, $b$, and $c$ are integers.
   A  $5x^2 - 37x + 14 = 0$
   C  $5x^2 + 37x + 14 = 0$
   B  $2x^2 + 9x - 35 = 0$
   D  $2x^2 - 9x - 35 = 0$

4. The current in one part of a series circuit is $3 - 2j$ amps.
The current in another part of the circuit is $2 + 4j$ amps.
Find the total amps in the circuit.
   F  $5 + 2j$
   H  $1 + 2j$
   G  $6 - 8j$
   J  $7j$

5. Solve $x^2 + 6x = -6$. If exact roots cannot be found, state the consecutive integers between which the roots are located.
   A  $-2, -3$
   C  between $-4$ and $-3$; between $-2$ and $-1$
   B  $-3$
   D  between $-5$ and $-4$; between $-2$ and $-1$

Part II

6. Solve $x^2 - 4x + 3 = 0$ by graphing.

7. Determine whether $f(x) = \frac{1}{2}x^2 - x - 9$
   has a maximum or a minimum value and find that value.

For Questions 8 and 9, solve each equation by factoring.

8. $x^2 - 7x = 18$
9. $4x^2 = x$

10. Simplify $\frac{5i}{3 - 5i}$. 
Write whether each sentence is true or false. If false, replace the underlined word or words to make a true sentence.

1. Two complex numbers of the form $a + bi$ and $a - bi$ are called imaginary units.
2. In $f(x) = 3x^2 - 2x + 5$, the linear term is 5.
3. $2x^2 + 3x - 4 \leq 0$ is an example of a quadratic equation.
4. The solutions of a quadratic equation are called its zeros.
5. The quadratic equation $y = 2(x + 3)^2 - 1$ is written in vertex form.
6. If a parabola opens upward, the $y$-coordinate of the vertex is the maximum value.
7. In $f(x) = -x^2 + 2x - 1$, the constant term is $-x^2$.
8. Pure imaginary numbers are square roots of negative real numbers.
9. The highest or lowest point on a parabola is called the vertex.
10. In the Quadratic Formula, the expression $b^2 - 4ac$ is called the quadratic term.

Define each term in your own words.

11. discriminant
12. standard form
Chapter 4 Test, Form 1

Write the letter for the correct answer in the blank at the right of each question.

1. Find the y-intercept for \( f(x) = -(x + 1)^2 \).
   - A 1
   - B -1
   - C \( x \)
   - D 0
   - 1. ___

2. What is the equation of the axis of symmetry of \( y = -3(x + 6)^2 + 12 \)?
   - F \( x = 2 \)
   - G \( x = -6 \)
   - H \( x = 6 \)
   - J \( x = -18 \)
   - 2. ___

3. The graph of \( f(x) = -2x^2 + x \) opens _____ and has a _____ value.
   - A down; maximum
   - B down; minimum
   - C up; maximum
   - D up; minimum
   - 3. ___

4. The related graph of a quadratic equation is shown at the right. Use the graph to determine the solutions of the equation.
   - F -2, 3
   - G 0, -6
   - H -3, 2
   - J 0, 2
   - 4. ___

5. The quadratic function \( f(x) = x^2 \) has _____.
   - A no zeros
   - B exactly one zero
   - C exactly two zeros
   - D more than two zeros
   - 5. ___

6. Solve \( x^2 - 3x - 10 = 0 \) by factoring.
   - F \( \{-5, 2\} \)
   - G \( \{-2, -5\} \)
   - H \( \{-2, 5\} \)
   - J \( \{-10, 1\} \)
   - 6. ___

7. Which quadratic equation has roots \(-2\) and \(3\)?
   - A \( x^2 + x + 6 = 0 \)
   - B \( x^2 - x - 6 = 0 \)
   - C \( x^2 - 6x + 1 = 0 \)
   - D \( x^2 + x - 6 = 0 \)
   - 7. ___

8. Simplify \( (5 + 2i)(1 + 3i) \).
   - F \( 5 + 6i \)
   - G \( -1 \)
   - H \( -1 + 17i \)
   - J \( 11 + 17i \)
   - 8. ___

9. ELECTRICITY The total impedance of a series circuit is the sum of the impedances of all parts of the circuit. A technician determined that the impedance of the first part of a particular circuit was \( 2 + 5j \) ohms. The impedance of the remaining part of the circuit was \( 3 - 2j \) ohms. What was the total impedance of the circuit?
   - A \( 5 + 3j \) ohms
   - B \( 5 + 7j \) ohms
   - C \( -1 + 7j \) ohms
   - D \( 16 + 11j \) ohms
   - 9. ___

10. To solve \( x^2 + 8x + 16 = 25 \) by using the Square Root Property, you would first rewrite the equation as _____.
    - F \( (x + 4)^2 = 25 \)
    - G \( (x + 4)^2 = 5 \)
    - H \( x^2 + 8x - 9 = 0 \)
    - J \( x^2 + 8x = 9 \)
    - 10. ___
11. Find the value of c that makes \( x^2 + 10x + c \) a perfect square.
   A 100   B 25   C 10   D 50   11.

12. The quadratic equation \( x^2 + 6x = 1 \) is to be solved by completing the square.
   Which equation would be the first step in that solution?
   F \( x^2 + 6x - 1 = 0 \)   H \( x^2 + 6x + 36 = 1 + 36 \)
   G \( x(x + 6) = 1 \)   J \( x^2 + 6x + 9 = 1 + 9 \)   12.

13. Find the exact solutions to \( x^2 - 3x + 1 = 0 \) by using the Quadratic Formula.
   A \( \frac{-3 + \sqrt{5}}{2} \)   B \( \frac{3 + \sqrt{13}}{2} \)   C \( \frac{-3 + \sqrt{13}}{2} \)   D \( \frac{3 + \sqrt{5}}{2} \)   13.

For Questions 14 and 15, use the value of the discriminant to determine the number and type of roots for each equation.

14. \( x^2 - 3x + 7 = 0 \)
   F 2 complex roots   H 2 real, irrational roots
   G 2 real, rational roots   J 1 real, rational root   14.

15. \( x^2 = 4x - 4 \)
   A 2 real, rational roots   C 1 real, rational root
   B 2 real, irrational roots   D no real roots   15.

16. What is the vertex of \( y = 2(x - 3)^2 + 6 \)?
   F (-2, -6)   G (3, -6)   H (-3, 6)   J (3, 6)   16.

17. What is the equation of the axis of symmetry of \( y = -3(x + 6)^2 + 1 \)?
   A \( x = 2 \)   B \( x = -6 \)   C \( x = -3 \)   D \( x = 6 \)   17.

18. Which quadratic function has its vertex at (2, 3) and passes through (1, 0)?
   F \( y = 2(x - 2)^2 + 3 \)   H \( y = -3(x + 2)^2 + 3 \)
   G \( y = -3(x - 2)^2 + 3 \)   J \( y = 2(x - 2)^2 - 3 \)   18.

19. Which quadratic inequality is graphed at the right?
   A \( y \geq (x + 1)^2 + 4 \)
   B \( y \leq -(x + 1)^2 + 4 \)
   C \( y \leq -(x - 1)^2 + 4 \)
   D \( y \leq -(x - 1)^2 - 4 \)   19.

20. Solve \( (x - 4)(x + 2) \leq 0 \).
   F \( \{x | x \leq -2 \text{ or } x \geq 4 \} \)
   G \( \{x | -2 \leq x \leq 4 \} \)
   H \( \{x | -4 \leq x \leq 2 \} \)
   J \( \{x | x = -2 \text{ or } x = 4 \} \)   20.

**Bonus** Find the x-intercepts and the y-intercept of the graph of \( y = 2(x - 4)^2 - 18 \).
   \( \boxed{B} \) _________________
4 Chapter 4 Test, Form 2A

Write the letter for the correct answer in the blank at the right of each question.

1. Identify the y-intercept and the axis of symmetry for the graph of
   \( f(x) = 10x^2 + 40x + 42 \).
   A 42; \( x = 4 \)  B 0; \( x = -4 \)  C 42; \( x = -2 \)  D -42; \( x = 2 \) 1.____

2. Identify the quadratic function graphed at the right.
   F \( f(x) = -x^2 - 2x \)  G \( f(x) = -x^2 + 2x \)  H \( f(x) = x^2 - 2x \)  J \( f(x) = -(x + 2)^2 \) 2.____

3. Determine whether \( f(x) = 4x^2 - 16x + 6 \) has a maximum or a minimum value and find that value.
   A minimum; -10  B minimum; 2  C maximum; -10  D maximum; 2 3.____

4. Solve \(-x^2 = 4x\) by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.
   F 4, 0  H -4, 0  G between -4 and 4  J -2, 4 4.____

5. Solve \( x^2 - 3x = 18 \) by factoring.
   A \{6\}  B \{-6, 3\}  C \{-9, 2\}  D \{-3, 6\} 5.____

6. Which quadratic equation has roots -2 and \( \frac{1}{5} \)?
   F \( x^2 + 4x + 4 = 0 \)  H \( 5x^2 - 9x - 2 = 0 \)  G \( 5x^2 + 9x - 2 = 0 \)  J \( 5x^2 - 11x + 2 = 0 \) 6.____

7. Simplify \((4 - 12i) - (-8 + 4i)\).
   A 12 - 8  B 28  C 12 - 16i  D 12 + 16i 7.____

8. Simplify \( \frac{4 - 2i}{7 + 3i} \).
   F \( \frac{11}{29} - \frac{13}{29}i \)  G \( \frac{11}{29} - \frac{14}{29}i \)  H \( \frac{13}{29} - \frac{17}{29}i \)  J \( \frac{11}{29} - \frac{13}{29}i \) 8.____

9. To solve \( 9x^2 - 12x + 4 = 49 \) by using the Square Root Property, you would first rewrite the equation as_____.
   A \( 9x^2 - 12x - 45 = 0 \)  C \( (3x - 2)^2 = 7 \)
   B \( (3x - 2)^2 = \pm 49 \)  D \( (3x - 2)^2 = 49 \) 9.____

10. Find the value of \( c \) that makes \( x^2 - 9x + c \) a perfect square.
    F \( \frac{81}{4} \)  G \( \frac{9}{2} \)  H \( -\frac{81}{4} \)  J 81 10.____
11. The quadratic equation \( x^2 - 8x = -20 \) is to be solved by completing the square. Which equation would be a step in that solution?
   \[ \text{A} \quad (x - 4)^2 = 4 \quad \text{C} \quad x^2 - 8x + 20 = 0 \]
   \[ \text{B} \quad x - 4 = \pm 2i \quad \text{D} \quad x^2 - 8x + 16 = -20 \]  
   \[ 11. \quad \]  

12. Find the exact solutions to \( 3x^2 = 5x - 1 \) by using the Quadratic Formula.
   \[ \text{F} \quad 5 \pm \sqrt{13} \quad \frac{6}{6} \quad \text{G} \quad 5 \pm \sqrt{13} \quad \frac{6}{6} \quad \text{H} \quad \frac{5 \pm \sqrt{37}}{2} \quad \text{J} \quad 5 \pm \sqrt{13} \quad \frac{6}{6} \]
   \[ 12. \quad \]  

For Questions 13 and 14, use the value of the discriminant to determine the number and type of roots for each equation.

13. \( 2x^2 - 7x + 9 = 0 \)
   \[ \text{A} \quad 2 \text{ real, rational} \quad \text{C} \quad 2 \text{ complex} \]
   \[ \text{B} \quad 2 \text{ real, irrational} \quad \text{D} \quad 1 \text{ real, rational} \]  
   \[ 13. \quad \]  

14. \( x^2 + 20 = 12x - 16 \)
   \[ \text{F} \quad 1 \text{ real, irrational} \quad \text{H} \quad 2 \text{ real, rational} \]
   \[ \text{G} \quad \text{no real} \quad \text{J} \quad 1 \text{ real, rational} \]  
   \[ 14. \quad \]  

15. Identify the vertex, axis of symmetry, and direction of opening for \( y = \frac{1}{2}(x - 8)^2 + 2 \).
   \[ \text{A} \quad (-8, 2); x = -8; \text{up} \quad \text{C} \quad (8, -2); x = 8; \text{up} \]
   \[ \text{B} \quad (-8, -2); x = -8; \text{down} \quad \text{D} \quad (8, 2); x = 8; \text{up} \]  
   \[ 15. \quad \]  

16. Which quadratic function has its vertex at \((-2, 7)\) and opens down?
   \[ \text{F} \quad y = -3(x + 2)^2 + 7 \quad \text{H} \quad y = (x - 2)^2 + 7 \]
   \[ \text{G} \quad y = -12(x + 2)^2 - 7 \quad \text{J} \quad y = -2(x - 2)^2 + 7 \]  
   \[ 16. \quad \]  

17. Write \( y = x^2 + 4x - 1 \) in vertex form.
   \[ \text{A} \quad y = (x - 2)^2 + 5 \quad \text{C} \quad y = (x + 2)^2 - 1 \]
   \[ \text{B} \quad y = (x + 2)^2 - 5 \quad \text{D} \quad y = (x + 2)^2 + 3 \]  
   \[ 17. \quad \]  

18. Write an equation for the parabola whose vertex is at \((-8, 4)\) and passes through \((-6, -2)\).
   \[ \text{F} \quad y = -\frac{3}{2}(x + 8)^2 + 4 \quad \text{H} \quad y = -\frac{1}{4}(x + 8)^2 + 4 \]
   \[ \text{G} \quad y = \frac{3}{2}(x + 6)^2 - 2 \quad \text{J} \quad y = -\frac{3}{2}(x - 8)^2 + 4 \]  
   \[ 18. \quad \]  

19. Which quadratic inequality is graphed at the right?
   \[ \text{A} \quad y \geq (x - 2)(x + 3) \quad \text{C} \quad y > (x + 2)(x - 3) \]
   \[ \text{B} \quad y > (x - 2)(x + 3) \quad \text{D} \quad y < (x + 2)(x - 3) \]  
   \[ 19. \quad \]  

20. Solve \( x^2 \geq 2x + 24 \).
   \[ \text{F} \quad \{x \mid -4 \leq x \leq 6\} \quad \text{H} \quad \{x \mid -6 \leq x \leq 4\} \]
   \[ \text{G} \quad \{x \mid x \leq -6 \text{ or } x \geq 4\} \quad \text{J} \quad \{x \mid x \leq -4 \text{ or } x \geq 6\} \]  
   \[ 20. \quad \]  

**Bonus** Write a quadratic equation with roots \( \pm \frac{i\sqrt{3}}{4} \). 

\[ \text{B:} \quad \]
Write the letter for the correct answer in the blank at the right of each question.

1. Identify the y-intercept and the axis of symmetry for the graph of \( f(x) = -3x^2 + 6x + 12. \)
   - A \( 2; x = -12 \)
   - B \( 12; x = 1 \)
   - C \( -2; x = 0 \)
   - D \( -12; x = -1 \)

2. Identify the quadratic function graphed at the right.
   - F \( f(x) = x^2 - 4x \)
   - G \( f(x) = -x^2 + 4x \)
   - H \( f(x) = -x^2 - 4x \)
   - J \( f(x) = -(x + 4)^2 \)

3. Determine whether \( f(x) = -5x^2 - 10x + 6 \) has a maximum or a minimum value and find that value.
   - A minimum; \(-1\)
   - B maximum; \(11\)
   - C maximum; \(-1\)
   - D minimum; \(11\)

4. Solve \( x^2 = 4x \) by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.
   - F \( -4, 0 \)
   - G \( 2, -4 \)
   - H between \(-4\) and \(4\)
   - J \( 0, 4 \)

5. Solve \( x^2 - 3x = 28 \) by factoring.
   - A \( \{-4, 7\} \)
   - B \( \{-14, 2\} \)
   - C \( \{-7, 4\} \)
   - D \( \{-2, 14\} \)

6. Which quadratic equation has roots \(7\) and \(\frac{-29}{3}\)?
   - F \( 2x^2 - 11x - 21 = 0 \)
   - G \( 3x^2 + 23x + 14 = 0 \)
   - H \( 3x^2 - 19x - 14 = 0 \)
   - J \( 2x^2 + 11x - 21 = 0 \)

7. Simplify \((15 - 13i) - (-1 + 17i)\).
   - A \( 16 - 30i \)
   - B \( 16 + 4i \)
   - C \( 16 + 30i \)
   - D \( 46 \)

8. Simplify \(\frac{1 + 2i}{2 - 3i}\).
   - F \( \frac{8}{7} + \frac{1}{7}i \)
   - G \( \frac{8}{7} + i \)
   - H \( -4 + 7i \)
   - J \( -\frac{4}{13} + \frac{7}{13}i \)

9. To solve \(4x^2 - 28x + 49 = 25\) by using the Square Root Property, you would first rewrite the equation as _____.
   - A \( (2x - 7)^2 = 25 \)
   - B \( (2x - 7)^2 = 5 \)
   - C \( (2x - 7)^2 = \pm 5 \)
   - D \( 4x^2 - 28x + 24 = 0 \)

10. Find the value of \(c\) that makes \(x^2 + 5x + c\) a perfect square trinomial.
    - F \( \frac{25}{16} \)
    - G \( \frac{5}{4} \)
    - H \( \frac{25}{4} \)
    - J \( \frac{25}{4} \)
11. The quadratic equation \( x^2 - 18x = -106 \) is to be solved by completing the square. Which equation would be a step in that solution?

A \((x - 9)^2 = 25\)  
B \(x^2 - 18x + 106 = 0\)  
C \(x - 9 = \pm 5i\)  
D \(x^2 - 18x + 81 = -106\)  

12. Find the exact solutions to \(2x^2 = 5x - 1\) by using the Quadratic Formula.

F \( \frac{-5 \pm \sqrt{17}}{4} \)  
G \( \frac{5 \pm \sqrt{17}}{4} \)  
H \( \frac{5 \pm \sqrt{33}}{4} \)  
J \( \frac{5 \pm \sqrt{17}}{2} \)

For Questions 13 and 14, use the value of the discriminant to determine the number and type of roots for each equation.

13. \(3x^2 - x - 12 = 0\)

A 2 complex roots  
B 1 real, rational root  
C 2 real, rational roots  
D 2 real, irrational roots  

14. \(x^2 + 10 = 3x - 3\)

F 2 complex roots  
G 1 real, rational root  
H 2 real, irrational roots  
J 2 real, rational roots  

15. Identify the vertex, axis of symmetry, and direction of opening for \(y = -8(x + 2)^2\).

A \((-8, -2); x = -8\) up  
B \((-2, 0); x = -2\) down  
C \((2, 0); x = 2\) down  
D \((-2, -8); x = -2\) down  

16. Which quadratic function has its vertex at \((-3, 5)\) and opens down?

F \(y = (x - 3)^2 + 5\)  
G \(y = -(x + 3)^2 + 5\)  
H \(y = (x + 3)^2 - 5\)  
J \(y = -(x - 3)^2 + 5\)

17. Write \(y = x^2 - 18x + 52\) in vertex form.

A \(y = (x - 9)^2 + 113\)  
B \(y = (x + 9)^2 - 29\)  
C \(y = (x - 9)^2 + 52\)  
D \(y = (x - 9)^2 - 29\)

18. Write an equation for the parabola whose vertex is at \((-5, 7)\) and passes through \((-3, -1)\).

F \(y = -\frac{1}{11}(x + 5)^2 + 7\)  
G \(y = -\frac{1}{2}(x + 5)^2 + 7\)  
H \(y = -2(x + 5)^2 + 7\)  
J \(y = -\frac{1}{2}(x - 5)^2 + 7\)

19. Which quadratic inequality is graphed at the right?

A \(y \leq (x - 3)(x + 1)\)  
B \(y > (x - 3)(x + 1)\)  
C \(y \geq (x + 3)(x - 1)\)  
D \(y > (x + 3)(x - 1)\)

20. Solve \(2x + 3 \geq x^2\).

F \(|x| - 1 \leq x \leq 3\)  
G \(|x| - 1 \leq x \leq 3\)  
H \(|x| - 1 \leq x \leq 3\)  
J \(|x| - 1 \leq x \leq 3\)  

Bonus  Write a quadratic equation with roots \(\pm \frac{i\sqrt{2}}{3}\).  

B: __________________
Chapter 4 Test, Form 2C

1. Graph \( f(x) = -5x^2 + 10x \), labeling the y-intercept, vertex, and axis of symmetry.

2. Determine whether \( f(x) = -3x^2 + 6x + 1 \) has a maximum or a minimum value and find that value.

3. Solve \( x^2 = 6x - 8 \) by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. Solve \( 5x^2 + 13x = 6 \) by factoring.

5. **GEOMETRY** The length of a rectangle is 7 inches longer than its width. If the area of the rectangle is 144 square inches, what are its dimensions?

6. **ELECTRICITY** The total impedance of a series circuit is the sum of the impedances of all parts of the circuit. Suppose that the first part of a circuit has an impedance of \( 6 - 5j \) ohms and that the total impedance of the circuit was \( 12 + 7j \) ohms. What is the impedance of the remainder of the circuit?

7. **ELECTRICITY** In an AC circuit, the voltage \( E \) (in volts), current \( I \) (in amps), and impedance \( Z \) (in ohms) are related by the formula \( E = I \cdot Z \). Find the current in a circuit with voltage \( 10 - 3j \) volts and impedance \( 4 + j \) ohms.

8. Write a quadratic equation with \( -6 \) and \( \frac{3}{4} \) as its roots. Write the equation in the form \( ax^2 + bx + c = 0 \), where \( a, b, \) and \( c \) are integers.

9. Solve \( x^2 + 6x + 9 = 25 \) by using the Square Root Property.
For Questions 10 and 11, solve each equation by completing the square.

10. \(x^2 + 4x - 9 = 0\)

11. \(2x^2 + 3x - 2 = 0\)

12. Find the exact solutions to \(5x^2 = 3x - 2\) by using the Quadratic Formula.

For Questions 13 and 14, find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

13. \(9x^2 - 12x + 4 = 0\)

14. \(4x^2 + 1 = 9x - 2\)

15. Identify the vertex, axis of symmetry, and direction of opening for \(y = -\frac{2}{3}(x + 5)^2 - 7\).

16. Write an equation for the parabola with vertex at \((2, -1)\) and \(y\)-intercept 5.

17. Write \(y = x^2 - 6x + 8\) in vertex form.

18. PHYSICS The height \(h\) (in feet) of a certain rocket \(t\) seconds after it leaves the ground is modeled by \(h(t) = -16t^2 + 48t + 15\). Write the function in vertex form and find the maximum height reached by the rocket.

19. Graph \(y < x^2 + 6x + 9\).

20. Solve \(2x^2 - 5x - 3 \geq 0\) algebraically.

Bonus Write a quadratic equation with roots \(\pm \sqrt[3]{7}\). Write the equation in the form \(ax^2 + bx + c = 0\), where \(a\), \(b\), and \(c\) are integers.
1. Graph \( f(x) = x^2 - 4x + 3 \), labeling the \( y \)-intercept, vertex, and axis of symmetry.

2. Determine whether \( f(x) = 5x^2 - 20x + 3 \) has a maximum or a minimum value and find that value.

3. Solve \( x^2 + 2x - 3 = 0 \) by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. Solve \( 3x^2 - x = 4 \) by factoring.

5. **GEOMETRY** The length of a rectangle is 10 inches longer than its width. If the area of the rectangle is 144 square inches, what are its dimensions?

6. **ELECTRICITY** The total impedance of a series circuit is the sum of the impedances of all parts of the circuit. Suppose that the first part of a circuit has an impedance of \( 7 + 4j \) ohms and that the total impedance of the circuit was \( 16 - 2j \) ohms. What is the impedance of the remainder of the circuit?

7. **ELECTRICITY** In an AC circuit, the voltage \( E \) (in volts), current \( I \) (in amps), and impedance \( Z \) (in ohms) are related by the formula \( E = I \cdot Z \). Find the impedance in a circuit with voltage \( 12 + 2j \) volts and current \( 3 + 5j \) amps.

8. Write a quadratic equation with \(-4\) and \(\frac{3}{2}\) as its roots. Write the equation in the form \( ax^2 + bx + c = 0 \), where \( a, b, \) and \( c \) are integers.
9. Solve $9x^2 + 12x + 4 = 6$ by using the Square Root Property.

Solve each equation by completing the square.

10. $x^2 - 8x + 14 = 0$

11. $3x^2 + x - 2 = 0$

12. Find the exact solutions to $2x^2 = 9x - 5$ by using the Quadratic Formula.

For Questions 13 and 14, find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

13. $25x^2 - 20x + 4 = 0$

14. $2x^2 + 10x + 9 = 2x$

15. Identify the vertex, axis of symmetry, and direction of opening for $y = -(x - 6)^2 - 5$.

16. Write an equation for the parabola with vertex at $(-4, 2)$ and $y$-intercept $-2$.

17. Write $y = x^2 + 4x + 8$ in vertex form.

18. PHYSICS The height $h$ (in feet) of a certain rocket $t$ seconds after it leaves the ground is modeled by $h(t) = -16t^2 + 64t + 12$. Write the function in vertex form and find the maximum height reached by the rocket.

19. Graph $y \geq x^2 - 4x + 4$.

20. Solve $2x^2 - 7x - 15 < 0$ algebraically.

**Bonus** Write a quadratic equation with roots $\pm \sqrt{5}/4$.

Write the equation in the form $ax^2 + bx + c = 0$, where $a$, $b$, and $c$ are integers.

B: ____________________
1. Graph \( f(x) = 3 + 3x^2 + 2x \), labeling the \( y \)-intercept, vertex, and axis of symmetry.

2. Determine whether \( f(x) = 1 - \frac{3}{5}x + \frac{3}{4}x^2 \) has a maximum or a minimum value and find that value.

3. **BUSINESS** Khalid charges $10 for a one-year subscription to his on-line newsletter. Khalid currently has 600 subscribers and he estimates that for each $1 decrease in the subscription price, he would gain 100 new subscribers. What subscription price will maximize Khalid’s income? If he charges this price, how much income should Khalid expect?

For Questions 4 and 5, solve each equation by graphing.
If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. \( 0.5x^2 + 9 = 4.5x \)

5. \( \frac{2}{3}x + 3 = \frac{1}{3}x^2 \)

6. Solve \( 18x^2 + 15 = 39x \) by factoring.

7. Write a quadratic equation with \(-\frac{2}{3}\) and 1.75 as its roots.
   Write the equation in the form \( ax^2 + bx + c = 0 \), where \( a, b, \) and \( c \) are integers.

8. Simplify \((5 - i) + (2 - 4i) - (3 + i)\).

9. Simplify \( \frac{2 - i\sqrt{5}}{2 + i\sqrt{5}} \).
10. Solve \(4x^2 - 2x + 0.25 = 1.44\) by using the Square Root Property.

For Questions 11 and 12, solve each equation by completing the square.

11. \(2x^2 - \frac{5}{2}x + 2 = 0\)

12. \(x^2 + 2.5x - 3 = 0.5\)

13. Find the exact solutions to \(\frac{1}{4}x^2 - 3x + 1 = 0\) by using the Quadratic Formula.

14. Find the value of the discriminant for \(3x(0.2x - 0.4) + 1 = 0.9\). Then describe the number and type of roots for the equation.

15. Find all values of \(k\) such that \(x^2 + kx + 1 = 0\) has two complex roots.

16. Write an equation of the parabola with equation \(y = -\frac{3}{5}(x - \frac{1}{2})^2 - \frac{5}{2}\), translated 4 units left and 2 units up. Then identify the vertex, axis of symmetry, and direction of opening of your function.

17. PHYSICS The height \(h\) (in feet) of a certain aircraft \(t\) seconds after it leaves the ground is modeled by \(h(t) = -9.1t^2 + 591.5t + 20,388.125\). Write the function in vertex form and find the maximum height reached by the aircraft.

18. Write an equation for the parabola that has the same vertex as \(y = \frac{1}{3}x^2 + 6x + \frac{83}{2}\) and \(x\)-intercept 1.

19. Graph \(y < -(x^2 + 2x) + 5.25\).

20. Solve \((x + \frac{7}{2})(x - 1)^2 \leq 0\).

**Bonus** Write a quadratic equation with roots \(\frac{-3 + 2i\sqrt{5}}{4}\).

Write the equation in the form \(ax^2 + bx + c = 0\), where \(a\), \(b\), and \(c\) are integers.

B: ____________________
Chapter 4 Extended-Response Test

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Mr. Moseley asked the students in his Algebra class to work in groups to solve \((x - 3)^2 = 25\), stating that each student in the first group to solve the equation correctly would earn five bonus points on the next quiz. Mi-Ling’s group solved the equation using the Square Root Property. Emilia’s group used the Quadratic Formula to find the solutions. In which group would you prefer to be? Explain your reasoning.

2. The next day, Mr. Moseley had his students work in pairs to review for their chapter exam. He asked each student to write a practice problem for his or her partner. Len wrote the following problem for his partner, Jocelyn: Write an equation for the parabola whose vertex is \((-3, -4)\), that passes through \((-1, 0)\), and opens down.
   b. How would you change Len’s problem?
   c. Make the change you suggested in part b and complete the problem.

3. a. Write a quadratic function in vertex form whose maximum value is 8.
   b. Write a quadratic function that transforms the graph of your function from part a so that it is shifted horizontally. Explain the change you made and describe the transformation that results from this change.

4. When asked to write \(f(x) = 2x^2 + 12x - 5\) in vertex form, Joseph wrote:
   \[
   f(x) = 2x^2 + 12x - 5 \\
   \text{Step 1} \quad f(x) = 2(x^2 + 6x) - 5 \\
   \text{Step 2} \quad f(x) = 2(x^2 + 6x + 9) - 5 + 9 \\
   \text{Step 3} \quad f(x) = 2(x + 3)^2 + 4
   \]
   Is Joseph’s answer correct? Explain your reasoning.

5. The graph of \(y = x^2 + 4x + 4\) is shown. Susan used this graph to solve three quadratic inequalities. Her three solutions are given below. Replace each \(\bullet\) with an inequality symbol \((<, >, \leq, \geq)\) so that each solution is correct. Explain your reasoning for each.
   a. The solution of \(x^2 + 4x + 4 \bullet 0\) is \(\{x | x < -2 \text{ or } x > -2\}\).
   b. The solution of \(x^2 + 4x + 4 \bullet 0\) is \(\varnothing\).
   c. The solution of \(x^2 + 4x + 4 \bullet 0\) is all real numbers.
Part 1: Multiple Choice

Instructions: Fill in the appropriate circle for the best answer.

1. If \( \frac{a}{b} = \frac{3}{2} \), then \( 8a \) equals which of the following?
   A. \( 16b \)  
   B. \( 12b \)  
   C. \( \frac{3b}{2} \)  
   D. \( \frac{8b}{3} \)  

2. 20% of 3 yards is how many fifths of 9 feet?
   F. 1  
   G. 6  
   H. 10  
   J. 15

3. If \( u > v \) and \( t > 0 \), which of the following are true?
   I. \( ut > vt \)  
   II. \( u + t > v + t \)  
   III. \( u - t > v - t \)
   A. I only  
   B. III only  
   C. I and II only  
   D. I, II, and III

4. Which of the following is the greatest?
   F. \( \frac{2}{3} \)  
   G. \( \frac{7}{9} \)  
   H. \( \frac{10}{15} \)  
   J. \( \frac{8}{11} \)

5. If \( 2a + 3b \) represents the perimeter of a rectangle and \( a - 2b \) represents its width, the length is ______.
   A. \( 7b \)  
   B. \( b \)  
   C. \( \frac{7b}{2} \)  
   D. \( 14b \)

6. In the figure, what is the area of the shaded region?
   F. 30 units²  
   G. 36 units²  
   H. 54 units²  
   J. 27 units²

7. Mr. Salazar rented a car for \( d \) days. The rental agency charged \( x \) dollars per day plus \( c \) cents per mile for the model he selected. When Mr. Salazar returned the car, he paid a total of \( T \) dollars. In terms of \( d, x, c, \) and \( T \), how many miles did he drive?
   A. \( T - (xd + c) \)  
   B. \( T - \frac{xd}{c} \)  
   C. \( \frac{T}{xd + c} \)  
   D. \( \frac{T - xd}{c} \)

8. If \( P(3, 2) \) and \( Q(7, 10) \) are the endpoints of the diameter of a circle, what is the area of the circle in square units?
   F. \( 2\sqrt{5}\pi \)  
   G. 80\pi  
   H. \( 4\sqrt{5}\pi \)  
   J. 20\pi

9. If \( (x - y)^2 = 100 \) and \( xy = 20 \), what is the value of \( x^2 + y^2 \)?
   A. 120  
   B. 140  
   C. 80  
   D. 60

10. The tenth term in the sequence 7, 12, 19, 28, . . . is ______.
    F. 124  
    G. 103  
    H. 57  
    J. 147
11. If \( t^2 + 6t = -9 \), what is the value of \( \left( t - \frac{1}{2} \right)^2 \)?
   A \(-3\)  B \(12 \frac{1}{4}\)  C \(6 \frac{1}{4}\)  D \(-12 \frac{1}{4}\)  

12. All four walls of a rectangular room that is 14 feet wide, 20 feet long, and 8 feet high, are to be painted. What is the minimum cost of paint if one gallon covers at most 130 square feet and the paint costs \$22\) per gallon?
   F \$92\)  G \$102\)  H \$110\)  J \$190\)

13. If \( i^2 = -1 \), then what is the value of \( i^{32} \)?
   A \(-1\)  B \(1\)  C \(-i\)  D \(i\)

14. Which of the following is the sum of both solutions of the equation \( x^2 + x - 42 = 0 \)?
   F \(13\)  G \(-1\)  H \(-13\)  J \(1\)

**Part 2: Gridded Response**

**Instructions:** Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate circle that corresponds to that entry.

15. The bar graph shows the distribution of votes among the candidates for senior class president. If 220 seniors voted in all, how many students voted for either Theo or Pam?

16. Find the median of \( x, 2x + 1, \frac{x}{2} - 13, 45, \) and \( x + 22 \) if the mean of this set of numbers is 83.
Part 3: Short Response

Instructions: Write your answer in the space provided.

17. Find the value of $12 + 36 \div 4 - (5 - 7)^2$.

18. Find the slope of the line that is parallel to the line with equation $3x + 4y = 10$.

19. Describe the system $2x - 3y = 21$ and $y - 5 = \frac{2}{3}x$ as consistent and independent, consistent and dependent, or inconsistent.

20. Find the coordinates of the vertices of the figure formed by the system of inequalities.
   \[
   \begin{align*}
   x &\geq -2 \\
y &\geq -2
   \end{align*}
   \]

21. Find the value of $\begin{vmatrix} 5 & 12 \\ -6 & 4 \end{vmatrix}$.

22. Solve $\begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 11 \\ -13 \end{bmatrix}$ by using inverse matrices.

23. Solve $2x^2 + 40 = 0$.

24. PHYSICS An object is thrown straight up from the top of a 100-foot platform at a velocity of 48 feet per second. The height $h(t)$ of the object $t$ seconds after being thrown is given by $h(t) = -16t^2 + 48t + 100$. Find the maximum height reached by the object and the time it takes to achieve this height.

25. Solve $x^2 = 2x + 3$ by graphing.

26. Solve $4x^2 - 4x = 24$ by factoring.

27. Find the value of the discriminant for $7x^2 + 5x + 1 = 0$. Then describe the number and type of roots for the equation.

28. Use $y = x^2 - 7x + 5$ for parts a–c.
   a. Write the equation in vertex form.
   b. Identify the vertex.
   c. Identify the axis of symmetry.
Study Guide and Intervention

Chapter 4

Anticipation Guide

Quadratic Functions and Relations

Step 1  Before you begin Chapter 4

Read each statement.

Step 2  A or D

1. All quadratic functions have a term with the variable to the second power. A

2. If the graph of the quadratic function \( y = ax^2 + c \) opens up then \( c < 0 \). D

3. A quadratic equation whose graph does not intersect the \( x \)-axis has no real solution. A

4. Since graphing shows the exact solutions to a quadratic equation, no other method is necessary for solving. D

5. If \( (x - 3)(x + 4) = 0 \), then either \( x = 3 \) or \( x = -4 \). A

6. An imaginary number contains \( i \), which equals the square root of \(-1\). A

7. A method called completing the square can be used to rewrite a quadratic expression as a perfect square. A

8. The quadratic formula can only be used for quadratic equations that cannot be solved by graphing or completing the square. D

9. The discriminant of a quadratic equation can be used to determine the direction the graph will open. D

10. The graph of \( y = 2x^2 \) is a dilation of the graph of \( y = x^2 \). A

11. The graph of \( y = x^2 + 2 \) will be two units to the right of the graph of \( y = x^2 \). D

12. The graph of a quadratic inequality containing the symbol \(<\) will be a parabola opening downward. D

Step 2  After you complete Chapter 4

Reread each statement and complete the last column by entering an A or a D.

Did any of your opinions about the statements change from the first column?

For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

Exercises

Complete parts a–c for each quadratic function.

a. Find the \( y \)-intercept, the equation of the axis of symmetry, and the \( x \)-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

1. \( f(x) = x^2 + 6x + 8 \)
   \[ x \quad f(x) \quad \text{vertex} \quad \text{graph} \]
   \[ -3 -2 -1 -4 \quad -1 \quad 0 \quad 0 \quad 3 \quad 0 \]

2. \( f(x) = -x^2 - 2x + 2 \)
   \[ x \quad f(x) \quad \text{vertex} \quad \text{graph} \]
   \[ -1 -0 -2 1 \quad 3 \quad 2 \quad 2 \quad -1 \]

3. \( f(x) = 2x^2 - 4x + 3 \)
   \[ x \quad f(x) \quad \text{vertex} \quad \text{graph} \]
   \[ 1 0 2 3 \quad 1 \quad 3 \quad 3 \quad 9 \]
Maximum and Minimum Values

The y-coordinate of the vertex of a quadratic function is the maximum value or minimum value of the function.

Maximum or Minimum Value

of a Quadratic Function

The graph of \( f(x) = ax^2 + bx + c \), where \( a \neq 0 \), opens up and has a minimum when \( a > 0 \). The graph opens down and has a maximum when \( a < 0 \).

**Example**

Determine whether each function has a maximum or minimum value, and find that value. Then state the domain and range of the function.

a. \( f(x) = 3x^2 - 6x + 7 \)

For this function, \( a = 3 \) and \( b = -6 \).

Since \( a > 0 \), the graph opens up, and the function has a minimum value.

The minimum value is the \( y \)-coordinate of the vertex. The \( x \)-coordinate of the vertex is \( -\frac{b}{2a} = -\frac{-6}{2 \cdot 3} = 1 \).

Evaluate the function at \( x = 1 \) to find the minimum value:

\( f(1) = 3(1)^2 - 6(1) + 7 = 4 \), so the minimum value of the function is 4. The domain is all real numbers. The range is all reals less than or equal to the minimum value, that is \( \{ f(x) | f(x) \geq 4 \} \).

b. \( f(x) = 100 - 2x - x^2 \)

For this function, \( a = -1 \) and \( b = -2 \).

Since \( a < 0 \), the graph opens down, and the function has a maximum value.

The maximum value is the \( y \)-coordinate of the vertex. The \( x \)-coordinate of the vertex is \( -\frac{b}{2a} = -\frac{-2}{2 \cdot -1} = 1 \).

Evaluate the function at \( x = -1 \) to find the maximum value:

\( f(-1) = 100 - 2(-1) - (-1)^2 = 101 \), so the maximum value of the function is 101. The domain is all real numbers. The range is all reals greater than or equal to the maximum value, that is \( \{ f(x) | f(x) \leq 101 \} \).

**Exercises**

Determine whether each function has a maximum or minimum value, and find that value. Then state the domain and range of the function.

1. \( f(x) = 2x^2 - 3x + 10 \)

min., \( \frac{17}{2} \); all reals; \( \{ f(x) | f(x) \leq \frac{17}{2} \} \)

2. \( f(x) = x^2 + 4x - 7 \)

min., \( -11 \); all reals; \( \{ f(x) | f(x) \geq -11 \} \)

3. \( f(x) = 3x^2 - 3x + 1 \)

min., \( \frac{1}{4} \); all reals; \( \{ f(x) | f(x) \geq \frac{1}{4} \} \)

4. \( f(x) = x^2 + 5x + 2 \)

min., \( -\frac{17}{2} \); all reals; \( \{ f(x) | f(x) \geq -\frac{17}{2} \} \)

5. \( f(x) = 20 + 6x - x^2 \)

max., \( 29 \); all reals; \( \{ f(x) | f(x) \leq 29 \} \)

6. \( f(x) = 4x^2 + x + 3 \)

min., \( \frac{1}{4} \); all reals; \( \{ f(x) | f(x) \geq \frac{1}{4} \} \)

7. \( f(x) = -x^2 - 4x + 10 \)

min., \( -14 \); all reals; \( \{ f(x) | f(x) \leq -14 \} \)

8. \( f(x) = x^2 - 10x + 5 \)

max., \( -20 \); all reals; \( \{ f(x) | f(x) \geq -20 \} \)

9. \( f(x) = -6x^2 + 12x + 21 \)

max., \( 27 \); all reals; \( \{ f(x) | f(x) \leq 27 \} \)

10. \( f(x) = 3x^2 \)

min., \( 0 \); all reals; \( \{ f(x) | f(x) \geq 0 \} \)

11. \( f(x) = x^2 + 1 \)

min., \( 1 \); all reals; \( \{ f(x) | f(x) \geq 1 \} \)

12. \( f(x) = -x^2 + 6x - 15 \)

max., \( -6 \); all reals; \( \{ f(x) | f(x) \leq -6 \} \)

13. \( f(x) = 2x^2 - 11 \)

min., \( -11 \); all reals; \( \{ f(x) | f(x) \leq -11 \} \)

14. \( f(x) = x^2 - 10x + 5 \)

max., \( 15 \); all reals; \( \{ f(x) | f(x) \geq 15 \} \)

15. \( f(x) = -2x^2 + 8x + 7 \)

min., \( -15 \); all reals; \( \{ f(x) | f(x) \geq -15 \} \)

Complete parts a–c for each quadratic function.

a. Find the \( y \)-intercept, the equation of the axis of symmetry, and the \( x \)-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

\[
\begin{align*}
1. f(x) &= -2x^2 \\
0; x &= 0 \\
4; x &= 2 \\
8; x &= 3 \\
2. f(x) &= x^2 - 4x + 4 \\
f(0) &= -2 \\
f(2) &= 0 \\
f(4) &= 8 \\
3. f(x) &= x^2 - 6x + 8 \\
f(0) &= 8 \\
f(2) &= 0 \\
f(4) &= 8 \\
\end{align*}
\]
Complete parts a–c for each quadratic function.

1. \( f(x) = x^2 - 8x + 15 \)
2. \( f(x) = -x^2 - 4x + 12 \)
3. \( f(x) = 2x^2 - 2x + 1 \)

**a.** Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

**b.** Make a table of values that includes the vertex.

**c.** Use this information to graph the function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>-1</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

Determine whether each function has a maximum or minimum value, and find that value. Then state the domain and range of the function.

4. \( f(x) = x^2 + 2x - 8 \)
   - Minimum: \(-9\); all reals; \( f(x) \leq -9 \)
5. \( f(x) = x^2 - 6x + 14 \)
   - Minimum: \(0\); all reals; \( f(x) \geq 0 \)
6. \( f(x) = -x^2 + 14x - 57 \)
   - Maximum: \(20\); all reals; \( f(x) \geq 20 \)

7. \( f(x) = 2x^2 + 4x - 6 \)
   - Minimum: \(-1\); all reals; \( f(x) \geq -1 \)
8. \( f(x) = -x^2 + 4x - 1 \)
   - Maximum: \(2\); all reals; \( f(x) \leq 2 \)
9. \( f(x) = -\frac{1}{8}x^2 + 8x - 24 \)
   - Minimum: \(2\); all reals; \( f(x) \leq 2 \)

10. \( f(x) = 2x^2 + 4x - 6 \)
    - Maximum: \(0\); all reals; \( f(x) \geq 0 \)

11. GRAVITATION From 4 feet above a swimming pool, Susan throws a ball upward with a velocity of 32 feet per second. The height \( h(t) \) of the ball \( t \) seconds after Susan throws it is given by \( h(t) = -16t^2 + 32t + 4 \). For \( t \geq 0 \), find the maximum height reached by the ball and the time that this height is reached.

12. HEALTH CLUBS Last year, the SportsTime Athletic Club charged \$20 to participate in an aerobics class. Seventy people attended the classes. The club wants to increase the class price this year. They expect to lose one customer for each \$1 increase in the price.
   - **a.** What price should the club charge to maximize the income from the aerobics classes? \$45
   - **b.** What is the maximum income the SportsTime Athletic Club can expect to make? \$2025

**Graphing Quadratic Functions**

1. **TRAJECTORIES** A cannonball is launched from a cannon on the wall of Fort Chambly, Quebec. If the path of the cannonball is traced on a piece of graph paper aligned so that the cannon is situated on the y-axis, the equation that describes the path is
   \[ y = -\frac{1}{1600}x^2 + \frac{1}{8}x + 20, \]
   where \( x \) is the horizontal distance from the cliff and \( y \) is the vertical distance above the ground in feet. How high above the ground is the cannon? 20 ft

2. **TICKETING** The manager of a symphony computes that the symphony will earn \(-40P + 1100\) dollars per concert if they charge \( P \) dollars for tickets. What ticket price should the symphony charge in order to maximize its profits? \$13.75

3. **ARCHES** An architect decides to use a parabolic arch for the main entrance of a science museum. In one of his plans, the top edge of the arch is described by the graph of \( y = -\frac{1}{8}x^2 + \frac{1}{8}x + 15 \). What are the coordinates of the vertex of this parabola? (5, 21.25)

4. **FRAMING** A frame company offers a line of square frames. If the side length of the frame is \( s \), then the area of the opening in the frame is given by the function \( a(s) = s^2 - 10s + 24 \).

5. **WALKING** Canal Street and Walker Street are perpendicular to each other. Evita is driving south on Canal Street and is currently 5 miles north of the intersection with Walker Street. Jack is at the intersection of Canal and Walker Streets and heading east on Walker. Jack and Evita are both driving 30 miles per hour.
   - **a.** When Jack is \( x \) miles east of the intersection, where is Evita? 5 - \( x \) mi north of the intersection
   - **b.** The distance between Jack and Evita is given by the formula \( \sqrt{x^2 + 5 - x^2} \). For what value of \( x \) are Jack and Evita at their closest? (Hint: Minimize the square of the distance.) \( x = 2.5 \)
   - **c.** What is the distance of closest approach? \( \frac{5\sqrt{2}}{2} \) mi
Finding the x-intercepts of a Parabola

As you know, if \( f(x) = ax^2 + bx + c \) is a quadratic function, the values of \( x \) that make \( f(x) \) equal to zero are \(-b \pm \sqrt{b^2 - 4ac} \over 2a\) and \(-b - \sqrt{b^2 - 4ac} \over 2a\).

The function \( f(x) \) has its maximum or minimum value when \( x = -b \over 2a \). The x-intercepts of the parabola, when they exist, are \(-b \pm \sqrt{b^2 - 4ac} \over 2a\) units to the left and right of the axis of symmetry.

Example

Find the vertex, axis of symmetry, and x-intercepts for \( f(x) = 5x^2 + 10x - 7 \).

Use \( x = -b \over 2a \).

\[ x = -\frac{b}{2a} = -1 \]  

The x-coordinate of the vertex is -1.

Substitute \( x = -1 \) in \( f(x) = 5x^2 + 10x - 7 \).

\[ f(-1) = 5(-1)^2 + 10(-1) - 7 = -12 \]  

The vertex is \((-1, -12)\).

The axis of symmetry is \( x = -\frac{b}{2a} \), or \( x = -1 \).

The x-coordinates of the x-intercepts are \( -1 \pm \sqrt{b^2 - 4ac} \over 2a \) and \( -1 - \sqrt{b^2 - 4ac} \over 2a \).

Exercises

Find the vertex, axis of symmetry, and x-intercepts for the graph of each function using \( x = -\frac{b}{2a} \).

1. \( f(x) = x^2 - 4x - 8 \) \( x = 2 \); 2. \( g(x) = -4x^2 - 8x + 3 \) \( x = -1 \); 3. \( y = -x^2 + 8x + 3 \) \( x = 4 \); 4. \( f(x) = 2x^2 + 6x + 5 \) \( x = -3 \); 5. \( A(x) = x^2 + 12x + 36 \) \( x = -6 \); 6. \( h(x) = -2x^2 + 2x - 6 \) \( x = \frac{1}{2} \).
4-2 Study Guide and Intervention (continued)

Solving Quadratic Equations by Graphing

Estimate Solutions Often, you may not be able to find exact solutions to quadratic equations by graphing. But you can use the graph to estimate solutions.

Example

Solve $x^2 - 2x - 2 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

The equation of the axis of symmetry of the related function is $x = \frac{-b}{2a} = \frac{2}{2} = 1$, so the vertex has $x$-coordinate 1. Make a table of values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-13</td>
</tr>
<tr>
<td>-2</td>
<td>-6</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-6</td>
</tr>
<tr>
<td>3</td>
<td>-13</td>
</tr>
</tbody>
</table>

The $x$-intercepts of the graph are between 2 and 3 and between 0 and -1. So one solution is between 2 and 3, and the other solution is between 0 and -1.

Exercises

Solve the equations. If exact roots cannot be found, state the consecutive integers between which the roots are located.

1. $x^2 - 4x + 2 = 0$
2. $x^2 + 6x + 6 = 0$
3. $x^2 + 4x + 2 = 0$
4. $x^2 + 2x + 4 = 0$
5. $2x^2 - 12x + 17 = 0$
6. $-\frac{1}{2}x^2 + x + \frac{5}{2} = 0$

- between 0 and 1; between 3 and 4
- between -2 and -1; between -5 and -4
- between -1 and 0; between -4 and -3
- between 3 and 4; between -2 and -1
- between 2 and 3; between 3 and 4
- between -2 and -1; between 3 and 4

4-2 Skills Practice

Solving Quadratic Equations by Graphing

Use the related graph of each equation to determine its solutions.

1. $x^2 + 2x - 3 = 0$
2. $-x^2 - 6x - 9 = 0$
3. $3x^2 + 4x + 3 = 0$
4. $x^2 - 6x + 5 = 0$
5. $-x^2 + 2x - 4 = 0$
6. $x^2 - 6x + 4 = 0$
7. $-x^2 - 4x = 0$
8. $-x^2 + 36 = 0$

- no real solutions
- between 1 and 5
- between 3 and 4
- between 0 and 1; between 5 and 6
- 0 and -4
4-2 Practice
Solving Quadratic Equations By Graphing

Use the related graph of each equation to determine its solutions.

1. \(-3x^2 + 3 = 0\)
   -1, 1
   no real solutions

2. \(3x^2 + x + 3 = 0\)
   no solutions

3. \(x^3 - 3x + 2 = 0\)
   1, 2

Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. \(-2x^2 - 6x + 5 = 0\)
   between 0 and 1; between -4 and -3

5. \(x^2 + 10x + 24 = 0\)
   -6, -4

6. \(2x^2 - x - 6 = 0\)
   -1.5, 2

7. \(-x^2 + x + 6 = 0\)
   3, -2

8. \(-x^2 + 5x - 8 = 0\)
   no such real numbers exist

9. GRAVITY Use the formula \(h(t) = vt - 16t^2\), where \(h(t)\) is the height of an object in feet, \(v\) is the objects initial upward velocity in feet per second, and \(t\) is the time in seconds.
   a. Marta throws a baseball with an initial upward velocity of 60 feet per second. How long after she releases the ball will it hit the ground? \(3.75 \text{ s}\)
   b. A volcanic eruption blasts a boulder upward with an initial velocity of 240 feet per second. How long will it take the boulder to hit the ground if it lands at the same elevation from which it was ejected? \(15 \text{ s}\)

4-2 Word Problem Practice
Solving Quadratic Equations by Graphing

1. TRAJECTORIES David threw a baseball into the air. The function of the height of the baseball in feet is \(h = 80t - 16t^2\), where \(t\) represents the time in seconds after the ball was thrown. Use this graph of the function to determine how long it took for the ball to fall back to the ground.

2. BRIDGES In 1895, a brick arch railway bridge was built on North Avenue in Baltimore, Maryland. The arch is described by the equation \(h = 9 - \frac{1}{50}x^2\), where \(h\) is the height in yards and \(x\) is the distance in yards from the center of the bridge. Graph this equation and describe, to the nearest yard, where the bridge touches the ground.

3. LOGIC Wilma is thinking of two numbers. The sum is 2 and the product is -24. Use a quadratic equation to find the two numbers. 6 and -4

4. RADIO TELESCOPES The cross-section of a large radio telescope is a parabola. The dish is set into the ground. The equation that describes the cross-section is \(d = \frac{32}{27}x^2 - \frac{4}{3}x - \frac{32}{3}\) where \(d\) gives the depth of the dish below ground level, what is the diameter of the dish? Solve by graphing.

5. BOATS The distance between two boats is \(d = \sqrt{t^2 - 10t + 35}\), where \(d\) is distance in meters and \(t\) is time in seconds.
   a. Make a graph of \(d^2\) versus \(t\).
   b. Do the boats ever collide? No
**Chapter 4**

4-2 Enrichment

**Graphing Absolute Value Equations**

You can solve absolute value equations in much the same way you solved quadratic equations. Graph the related absolute value function for each equation using a graphing calculator. Then use the ZERO feature in the CALC menu to find its real solutions, if any. Recall that solutions are points where the graph intersects the x-axis.

For each equation, make a sketch of the related graph and find the solutions rounded to the nearest hundredth.

1. \(|x + 5| = 0\)  
2. \(|4x - 3| + 5 = 0\)  
3. \(|x - 7| = 0\)  

- \(5\)  
- No solutions  
- \(7\)

4. \(|x + 3| - 8 = 0\)  
5. \(|x + 3| + 6 = 0\)  
6. \(|x - 2| - 3 = 0\)  

- \(-11, 5\)  
- \(-9, 3\)  
- \(-1, 5\)

7. \(|3x + 4| = 2\)  
8. \(|x + 12| = 10\)  
9. \(|x - 3| = 0\)  

- \(-2, -\frac{2}{3}\)  
- \(-22, -2\)  
- \(-3, 3\)

10. Explain how solving absolute value equations algebraically and finding zeros of absolute value functions graphically are related.

Sample answer: Values of x when solving algebraically are the x-intercepts (or zeros) of the function when graphed.

4-3 Study Guide and Intervention

**Solving Quadratic Equations by Factoring**

**Factored Form** To write a quadratic equation with roots \(p\) and \(q\), let \((x - p)(x - q) = 0\). Then multiply using FOIL.

**Example** Write a quadratic equation in standard form with the given roots.

a. \(-3, -5\)  

\((x - (-3))(x - (-5)) = 0\)  

\(x^2 + 2x - 15 = 0\)  

b. \(\frac{7}{8}, \frac{3}{8}\)  

\((x - \frac{7}{8})(x - \frac{3}{8}) = 0\)  

\(x^2 - \frac{10}{8}x + \frac{21}{64} = 0\)

**Exercises**

Write a quadratic equation in standard form with the given root(s).

1. \(3, -4\)  
2. \(-2, -3\)  
3. \(1, 9\)

\(x^2 + x - 12 = 0\)  
\(x^2 + 10x + 16 = 0\)  
\(x^2 - 10x + 9 = 0\)

4. \(-5\)  
5. \(10, 7\)  
6. \(-2, 5\)

\(x^2 + 10x + 25 = 0\)  
\(x^2 - 17x + 70 = 0\)  
\(x^2 - 13x - 30 = 0\)

7. \(-\frac{1}{2}, 5\)  
8. \(2, \frac{3}{2}\)  
9. \(-\frac{7}{2}, \frac{3}{4}\)

\(3x^2 - 14x - 5 = 0\)  
\(3x^2 - 8x + 4 = 0\)  
\(4x^2 + 25x - 21 = 0\)

10. \(\frac{3}{5}, \frac{5}{2}\)  
11. \(-\frac{4}{9}, -1\)  
12. \(9, \frac{1}{6}\)

\(5x^2 - 17x + 6 = 0\)  
\(9x^2 + 13x + 4 = 0\)  
\(6x^2 - 55x + 9 = 0\)

13. \(-\frac{2}{3}, \frac{2}{3}\)  
14. \(\frac{3}{4}, 4\)  
15. \(\frac{1}{3}, \frac{3}{4}\)

\(9x^2 - 4 = 0\)  
\(8x^2 - 6x - 5 = 0\)  
\(35x^2 - 22x + 3 = 0\)

16. \(\frac{7}{8}, \frac{3}{2}\)  
17. \(\frac{3}{2}, 4\)  
18. \(\frac{1}{6}, \frac{1}{6}\)

\(16x^2 - 42x - 49 = 0\)  
\(8x^2 - 10x + 3 = 0\)  
\(48x^2 - 14x + 1 = 0\)
For any real numbers \( a \) and \( b \), if \( ab = 0 \), then either \( a = 0 \) or \( b = 0 \), or both \( a \) and \( b \) are zero.

**Example** Solve each equation by factoring.

a. \( 3x^2 = 15x \)

\[ 3x^2 - 15x = 0 \]

Subtract 15 from both sides.

\[ 3x(x - 5) = 0 \]

Factor the binomial.

\[ x = 0 \quad \text{or} \quad x = 5 \]

The solution set is \( \{0, 5\} \).

b. \( 4x^2 - 5x = 21 \)

\[ 4x^2 - 5x - 21 = 0 \]

Original equation

Subtract 21 from both sides.

\[ (x - 7)(4x + 3) = 0 \]

Factor the binomial.

\[ x = \frac{7}{4} \quad \text{or} \quad x = -\frac{3}{4} \]

The solution set is \( \left\{ \frac{7}{4}, -\frac{3}{4} \right\} \).

**Exercises** Solve each equation by factoring.

1. \( x^2 - 2x = 0 \)

\[ x(x - 2) = 0 \]

\[ x = 0 \quad \text{or} \quad x = 2 \]

2. \( x^2 = 7x \)

\[ x(x - 7) = 0 \]

\[ x = 0 \quad \text{or} \quad x = 7 \]

3. \( 20x^2 = -25x \)

\[ 20x - 25 = 0 \]

\[ 5x = 5 \]

\[ x = 1 \]

4. \( 6x^2 = 7x \)

\[ 6x^2 - 7x = 0 \]

\[ x(6x - 7) = 0 \]

\[ x = 0 \quad \text{or} \quad x = \frac{7}{6} \]

5. \( 2x^2 - x - 30 = 0 \)

\[ (x - 6)(2x + 5) = 0 \]

\[ x = 6 \quad \text{or} \quad x = -\frac{5}{2} \]

6. \( 4x^2 + 27x - 7 = 0 \)

\[ (4x - 1)(x + 7) = 0 \]

\[ x = \frac{1}{4} \quad \text{or} \quad x = -7 \]

7. \( 13x^2 - 8x + 1 = 0 \)

\[ (13x - 1)(x + 1) = 0 \]

\[ x = \frac{1}{13} \quad \text{or} \quad x = -1 \]

8. \( 12x^2 + 25x + 100 = 0 \)

\[ (4x + 5)(3x + 4) = 0 \]

\[ x = -\frac{5}{4} \quad \text{or} \quad x = -\frac{4}{3} \]

9. \( 17x^2 + 2x - 3 = 0 \)

\[ (17x - 3)(x + 1) = 0 \]

\[ x = \frac{3}{17} \quad \text{or} \quad x = -1 \]

10. \( 19x^2 - 6x + 5 = 0 \)

\[ (19x - 1)(x + 5) = 0 \]

\[ x = \frac{1}{19} \quad \text{or} \quad x = -5 \]

11. \( 21x^2 - 4x - 21 = 0 \)

\[ (7x - 3)(3x + 7) = 0 \]

\[ x = \frac{3}{7} \quad \text{or} \quad x = -\frac{7}{3} \]

12. \( c^2 - 100 \)

\[ (c + 10)(c - 10) \]

Solve each equation by factoring.

13. \( x^2 = 64 \)

\[ \{8, -8\} \]

14. \( x^2 = 100 \)

\[ \{10, -10\} \]

15. \( x^2 - 3x + 2 = 0 \)

\[ \{1, 2\} \]

16. \( x^2 - 4x + 3 = 0 \)

\[ \{1, 3\} \]

17. \( x^2 - 2x - 3 = 0 \)

\[ \{-1, 3\} \]

18. \( x^2 - 3x - 10 = 0 \)

\[ \{-5, 2\} \]

19. \( 20x^2 + 9x = 0 \)

\[ \{0, 9\} \]

20. \( x^2 + 9x = 0 \)

\[ \{-9, 0\} \]

21. \( 2x^2 + 3x - 10 = 0 \)

\[ \{-5, 2\} \]

22. \( 4x^2 + 5x = 0 \)

\[ \{-5, 0\} \]

23. \( 3x^2 - 13x - 10 = 0 \)

\[ \{-\frac{2}{3}, 5\} \]

24. \( 25 \), \( 16 \) or \( -16 \), \(-17\)
4.3 Practice

Solving Quadratic Equations by Factoring

Write a quadratic equation in standard form with the given root(s).

1. $2, 0.3$
   
   $x^2 - 2x = 0$
   $x^2 - 3x - 40 = 0$

2. $4, -7, -8$
   
   $x^2 + 15x + 56 = 0$
   $x^2 + x - 12 = 0$

3. $7, 0.25$
   
   $2x^2 - 3x + 1 = 0$
   $3x^2 - 7x + 2 = 0$
   $2x^2 + 7x = 0$

Factor each polynomial.

4. $10, 3, -5$
   
   $x^2 + 3x - 54 = 0$
   $2(4x - 3)(x + 1) (c - 7)(c + 7)$

5. $13, x + 8$
   
   $(x + 2)(x^2 - 2x + 4)$
   $(4r + 13)(4r - 13) (b^2 + 9)(b + 3)(b - 3)$

Solve each equation by factoring.

6. $x^2 - 4x - 12 = 0$
   
   $x^2 - 16x + 64 = 0 (8)$

7. $x^2 - 6x + 8 = 0 (2, 4)$
   
   $19x^2 + 3x + 2 = 0 (-2, -1)$

8. $x^2 - 4x = 0 (0, 4)$
   
   $21.7x^4 = 4x (0, 4)$

9. $22.10x^2 = 9x$
   
   $23. x^2 = 2x + 99 (-9, 11)$

10. $24. x^2 + 12x = -36 (-6)$
    
    $25. 5x^2 - 35x + 60 = 0 (3, 4)$

11. $26. 36x^2 = 25 \left( \frac{5}{6} \right) - 5 \left( \frac{5}{6} \right)$
    
    $27. 2x^2 - 8x - 90 = 0 (9, -5)$

12. $28. NUMBERS THEORY$ Find two consecutive even positive integers whose product is 624.
    
    $29. NUMBERS THEORY$ Find two consecutive odd positive integers whose product is 323.
    
    $30. GEOMETRY$ The length of a rectangle is 2 feet more than its width. Find the dimensions of the rectangle if its area is 63 square feet. 7 ft by 9 ft
    
    $31. PHOTOGRAPHY$ The length and width of a 6-inch by 8-inch photograph are reduced by the same amount to make a new photograph whose area is half that of the original. By how many inches will the dimensions of the photograph have to be reduced? 2 in.
Using Patterns to Factor

Study the patterns below for factoring the sum and the difference of cubes.

\[ a^3 + b^3 = (a + b)(a^2 - ab + b^2) \]
\[ a^3 - b^3 = (a - b)(a^2 + ab + b^2) \]

This pattern can be extended to other odd powers. Study these examples.

**Example 1**
Factor \( a^3 + b^3 \).

Extend the first pattern to obtain \( a^3 + b^3 = (a + b)(a^2 - ab + b^2) \).

Check: \( a + b(a^2 - ab + b^2) = a^3 + b^3 \).

\[ a^3 + b^3 = (a + b)(a^2 - ab + b^2) \]

**Example 2**
Factor \( a^3 - b^3 \).

Extend the second pattern to obtain \( a^3 - b^3 = (a - b)(a^2 + ab + b^2) \).

Check: \( a - b(a^2 + ab + b^2) = a^3 - b^3 \).

\[ a^3 - b^3 = (a - b)(a^2 + ab + b^2) \]

In general, if \( n \) is an odd integer, when you factor \( a^n + b^n \) or \( a^n - b^n \), one factor will be either \( a + b \) or \( a - b \), depending on the sign of the original expression. The other factor will have the following properties:

- The first term will be \( a^{n-1} \) and the last term will be \( b^{n-1} \).
- The exponents of \( a \) will decrease by 1 as you go from left to right.
- The exponents of \( b \) will increase by 1 as you go from left to right.
- The degree of each term will be \( n - 1 \).
- If the original expression was \( a^n + b^n \), the terms will alternately have + and - signs.
- If the original expression was \( a^n - b^n \), the terms will all have + signs.

Use the patterns above to factor each expression.

1. \( x^3 + 8 \)\( \rightarrow \) \( (x + 2)(x^2 - 2x + 4) \)
2. \( x^3 - 27 \)\( \rightarrow \) \( (x - 3)(x^2 + 3x + 9) \)
3. \( x^3 + 27 \)\( \rightarrow \) \( (x + 3)(x^2 - 3x + 9) \)
4. \( x^3 - 64 \)\( \rightarrow \) \( (x - 4)(x^2 + 4x + 16) \)

**Exercises**

Factor each polynomial.

1. \( x^3 + 2x^2 - 15x + 25 = (x - 5)(x^2 + 5x - 5) \)
2. \( x^3 - 27 = (x - 3)(x^2 + 3x + 9) \)
3. \( x^3 - 8 = (x - 2)(x^2 + 2x + 4) \)
4. \( x^3 + 27 = (x + 3)(x^2 - 3x + 9) \)
5. \( x^3 - 1 = (x - 1)(x^2 + x + 1) \)
6. \( x^3 + 8 = (x + 2)(x^2 - 2x + 4) \)
7. \( x^3 - 125 = (x - 5)(x^2 + 5x + 25) \)
8. \( x^3 + 27 = (x + 3)(x^2 - 3x + 9) \)
9. \( x^3 - 64 = (x - 4)(x^2 + 4x + 16) \)
10. \( x^3 - 125 = (x - 5)(x^2 + 5x + 25) \)

**Graphing Calculator Activity**

**Using Tables to Factor by Grouping**

The TABLE feature of a graphing calculator can be used to help factor a polynomial of the form \( ax^2 + bx + c \). (The same problems can be solved with the Lists and Spreadsheet application on the TI-Nspire.)

**Example 1**
Factor \( 10x^3 - 43x^2 + 28x \) by grouping.

Make a table of the negative factors of 10 - 28 or 280. Look for a pair of factors whose sum is -43.

Enter the equation \( y = \frac{280}{x} \) in Y1 to find the factors of 280. Then, find the sum of the factors using \( y = \frac{280}{x} \) in Y2. Set up the table to display the negative factors of 280 by setting \( \Delta \text{Th1} = -1 \).

Examine the results.

**Example 2**
Factor \( 12x^2 - 7x - 12 \).

Look at the factors of 12 - 12 or 144 for a pair with a sum of -7.

Enter an equation to determine the factors in Y1 and an equation to find the sum of the factors in Y2. Examine the table to find a sum of -7.

**Exercises**

Factor each polynomial.

1. \( x^3 - 20x + 96 = (x - 4)(x^2 + 4x - 24) \)
2. \( x^3 - 27x^2 + 27x = x(x^2 - 27x + 27) \)
3. \( x^3 - 10x^2 + 25x = x(x^2 - 10x + 25) \)
4. \( x^3 + 3x^2 - 4x - 12 = (x + 3)(x^2 - 4x - 4) \)
5. \( x^3 - 27x^3 + 27x = x(x^2 - 27x + 27) \)
6. \( x^3 - 125x^2 + 625x = x(x^2 - 125x + 625) \)
7. \( x^3 - 1000x^2 + 1000x = x(x^2 - 1000x + 1000) \)
8. \( x^3 + 3x^2 - 4x - 4 = (x + 3)(x^2 - 4x - 4) \)
9. \( x^3 - 27x^2 + 27x = x(x^2 - 27x + 27) \)
10. \( x^3 - 1000x^2 + 1000x = x(x^2 - 1000x + 1000) \)
4-4 Study Guide and Intervention

Complex Numbers

Pure Imaginary Numbers A square root of a number \( n \) is a number whose square is \( n \). For nonnegative real numbers \( a \) and \( b \), \( \sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \) and \( \sqrt{a^2} = |a| \), \( b \neq 0 \).

- The imaginary unit \( i \) is defined to have the property that \( i^2 = -1 \).
- Simplified square root expressions do not have radicals in the denominator, and any number remaining under the square root has no perfect square factor other than 1.

**Example 1**

a. Simplify \( \sqrt{-48} \).
   \[ \sqrt{-48} = \sqrt{16 \cdot (-3)} \]
   \[ = \sqrt{16} \cdot \sqrt{-3} \cdot \sqrt{-1} \]
   \[ = 4 \cdot \sqrt{3} \cdot i \]

b. Simplify \( \sqrt{-63} \).
   \[ \sqrt{-63} = \sqrt{-1} \cdot \sqrt{9} \cdot \sqrt{-9} \]
   \[ = 3i \sqrt{7} \]

**Example 2**

a. Simplify \(-3i \cdot 4i\).
   \[ -3i \cdot 4i = -12i^2 \]
   \[ = -12 (-1) \]
   \[ = 12 \]

b. Simplify \( \sqrt{-3} \cdot \sqrt{-15} \).
   \[ \sqrt{-3} \cdot \sqrt{-15} = \sqrt{3} \cdot i \cdot \sqrt{15} \]
   \[ = \sqrt{45} \]
   \[ = 3 \sqrt{5} \]

**Example 3** Solve \( x^2 + 3 = 0 \).

\( x^2 + 5 = 0 \)  
\( x^2 = -5 \)  
\( x = \pm \sqrt{-5} \)  
Square Root Property

Solve each equation.

5. \( 5x^2 + 45 = 0 \)  
6. \( 4x^2 + 24 = 0 \)  
7. \( -9x^2 = 9 \)  
8. \( 7x^2 + 84 = 0 \)

**Exercises**

Simplify.

1. \( \sqrt{-72} \)  
2. \( 2\sqrt{24} \)  
3. \( \sqrt{-84} \)  
4. \( (2 + i)(2 - i) \)

Solve each equation.

5. \( 5x^2 + 45 = 0 \)  
6. \( 4x^2 + 24 = 0 \)  
7. \( -9x^2 = 9 \)  
8. \( 7x^2 + 84 = 0 \)

**Answers**

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Chapter 4

4-4 Skills Practice

Complex Numbers

Simplify.
1. $\sqrt{9} \quad 3\sqrt{11} + 2\sqrt{2}$
2. $\sqrt{\frac{27}{19}} \quad \frac{3\sqrt{3}}{7}$
3. $\sqrt{2x^2 \cdot 13xy}$
4. $\sqrt{-100x^2} \quad 6i \cdot x \cdot \sqrt{3x}$
5. $\sqrt{-81x^2} \quad 9x \cdot \sqrt{x} ^3$
6. $\sqrt{25 \cdot 26 - 23 \cdot 2} - 23\sqrt{2}$
7. $(3i)(-2i)(-3i) \quad 30i$
8. $i^8 - i$
9. $2^n i$
10. $(7 - 8i) + (-12 - 4i) \quad -5 - 12i$
11. $(-3 + 5i) + (18 - 7i) \quad 15 - 2i$
12. $(10 - 4i) - (7 + 3i) \quad 3 - 7i$
13. $(7 - 6i) \cdot 2 - 3i \quad -4 - 33i$
14. $(3 + 4i) \cdot (4 - 4i) \quad 25$
15. $\frac{8 - 9i}{3i} - 2 - \frac{9}{3} i$
16. $\sqrt{\frac{34}{4} \cdot \frac{3}{10} + \frac{3}{5}}$

Solve each equation.
17. $3x^3 + 3 = 0 \quad \pm i$
18. $5x^2 + 125 = 0 \quad \pm 5i$
19. $4x^2 + 20 = 0 \quad \pm i\sqrt{5}$
20. $-x^2 - 16 = 0 \quad \pm 4i$
21. $x^2 + 18 = 0 \quad \pm 3\sqrt{2}$
22. $8x^2 + 96 = 0 \quad \pm 2i\sqrt{3}$

Find the values of $\ell$ and $m$ that make each equation true.
23. $20 - 12i = 5\ell + (4m)i \quad 4, -3$
24. $\ell - 16i = 3 - (2m)i \quad 3, 8$
25. $(4 + \ell) + (2m)i = 9 + 1i \quad 5, 7$
26. $(3 - m) + (7\ell - 14)i = 1 + 7i \quad 3, 2$

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Chapter 4

4-4 Word Problem Practice

Complex Numbers

1. SIGN ERRORS Jennifer and Jessica come up with different answers to the same problem. They had to multiply (4 + i)(4 - i) and give their answer as a complex number. Jennifer claims that the answer is 15 and Jessica claims that the answer is 17. Who is correct? Explain.

Jennifer is correct; (4 + i)(4 - i) = 16 + 4i - 4i - i^2 = 16 + 1 = 17.

2. COMPLEX CONJUGATES You have seen that the product of complex conjugates is always a real number. a + bi and a - bi are complex conjugates and their sum is 2a, which is real.

3. PYTHAGOREAN TRIPLES If three integers a, b, and c satisfy a^2 + b^2 = c^2, then they are called a Pythagorean triple. Suppose that a, b, and c are a Pythagorean triple. Show that the real and imaginary parts of a + bi and a - bi are complex conjugates and that their sum is 2a, which is real.

(a + bi)^2 = a^2 - b^2 + 2abi; a^2 - b^2 and 2ab are integers and (a^2 - b^2)^2 + (2ab)^2 = a^4 + 2a^2b^2 + b^4 = (a^2 + b^2)^2 = (c^2)^2, so a^2 + b^2 = c^2 as desired.

4. ROTATIONS Complex numbers can be used to perform rotations in the plane. For example, if (x, y) are the coordinates of a point in the plane, then the real and imaginary parts of (x + iy) are the horizontal and vertical coordinates of the 90° counterclockwise rotation of (x, y) about the origin. What are the real and imaginary parts of i(x + iy)?

The real part is -x and imaginary part is y.

5. ELECTRICAL ENGINEERING Alternating current (AC) in an electrical circuit can be described by complex numbers. In any electrical circuit, Z, the impedance in the circuit, is related to the voltage V and the current I by the formula Z = \( \frac{V}{I} \). The standard electrical voltage in Europe is 220 volts, so in these problems use V = 220.

a. Find the impedance in a standard European circuit if the current is 22 - 11i amps.

b. Find the current in a standard European circuit if the impedance is 10 - 5i watts.

c. Find the impedance in a standard European circuit if the current is 20i amps.

Exercises

For each of the following complex numbers, find the absolute value and multiplicative inverse.

1. 2i; \( \frac{1}{2} \)
2. -4 - 3i; \( \frac{-4 + 3i}{25} \)
3. 12 - 5i; 13; \( \frac{12 + 5i}{169} \)

4. 5 - 12i; \( \frac{5 + 12i}{169} \)
5. 1 + i \( \sqrt{2} \); \( \frac{1 - i}{2} \)
6. \( \sqrt{3} - i \); 2; \( \frac{\sqrt{3} + i}{4} \)

7. \( \frac{\sqrt{3}}{3} - \frac{i}{3} \)
8. \( \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} i \)
9. \( \frac{1}{2} + \frac{\sqrt{3}}{2} i \)

4-4 Enrichment

Conjugates and Absolute Value

When studying complex numbers, it is often convenient to represent a complex number by a single variable. For example, we might let z = x + iy. We denote the conjugate of z by \( \overline{z} \). Thus, \( \overline{z} = x - iy \).

We can define the absolute value of a complex number as follows.

\[ |z| = |x + iy| = \sqrt{x^2 + y^2} \]

There are many important relationships involving conjugates and absolute values of complex numbers.

Example 1. Show that \( \overline{z \overline{f}} = zf \) for any complex number z.

Let \( z = x + iy \). Then,

\[ \overline{z} = (x + iy)(x - iy) = x^2 + y^2 = \sqrt{x^2 + y^2}^2 = |z|^2 \]

Example 2. Show that \( \frac{z}{|z|} \) is the multiplicative inverse for any nonzero complex number z.

We know \( |\overline{z}| = x \). If \( z \neq 0 \), then we have \( \frac{1}{|z|} = \frac{1}{x} \).

Thus, \( \frac{z}{|z|} \) is the multiplicative inverse of z.

Exercises

For each of the following complex numbers, find the absolute value and multiplicative inverse.

1. 2i; \( \frac{1}{2} \)
2. -4 - 3i; \( \frac{-4 + 3i}{25} \)
3. 12 - 5i; 13; \( \frac{12 + 5i}{169} \)

4. 5 - 12i; \( \frac{5 + 12i}{169} \)
5. 1 + i \( \sqrt{2} \); \( \frac{1 - i}{2} \)
6. \( \sqrt{3} - i \); 2; \( \frac{\sqrt{3} + i}{4} \)

7. \( \frac{\sqrt{3}}{3} - \frac{i}{3} \)
8. \( \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} i \)
9. \( \frac{1}{2} + \frac{\sqrt{3}}{2} i \)
Completing the Square
Square Root Property

Use the Square Root Property to solve a quadratic equation that is in the form "perfect square trinomial = constant."

**Example**

Solve each equation by using the Square Root Property. Round to the nearest hundredth if necessary.

- **a.** \( x^2 - 8x + 16 = 25 \)
  - \( x^2 - 8x + 16 = 25 \)
  - \( x - 4 = \sqrt{25} \) or \( x - 4 = -\sqrt{25} \)
  - \( x = 5 + 4 = 9 \) or \( x = -5 + 4 = -1 \)
  - The solution set is \( \{9, -1\} \).

- **b.** \( 4x^2 - 20x + 25 = 32 \)
  - \( 4x^2 - 20x + 25 = 32 \)
  - \( 2x^2 - 5x = 5/2 \)
  - \( 2x - 5 = \sqrt{2} \) or \( 2x - 5 = -\sqrt{2} \)
  - \( x = \frac{5 \pm \sqrt{2}}{2} \)
  - The solution set is \( \{5 \pm \sqrt{2}/2\} \).

**Exercises**

Solve each equation by using the Square Root Property. Round to the nearest hundredth if necessary.

1. \( x^2 + 12x + 1 = 49 \)  
   - \( x^2 + 12x + 1 = 49 \)
   - \( x = \frac{-12 \pm \sqrt{144}}{2} \)
   - \( x = -12 \pm 12 \)
   - The solution set is \( \{-24, 0\} \).

2. \( x^2 - 7x + 10 = 0 \)  
   - \( x^2 - 7x + 10 = 0 \)
   - \( x = \frac{7 \pm \sqrt{49 - 40}}{2} \)
   - \( x = \frac{7 \pm \sqrt{9}}{2} \)
   - \( x = \frac{7 \pm 3}{2} \)
   - The solution set is \( \{2, 5\} \).

3. \( x^2 - 4x + 4 = 9 \)  
   - \( x^2 - 4x + 4 = 9 \)
   - \( x = \frac{4 \pm \sqrt{16 - 16}}{2} \)
   - \( x = \frac{4 \pm 0}{2} \)
   - The solution set is \( \{2\} \).

4. \( 3x^2 + 12x + 11 = 18 \)  
   - \( 3x^2 + 12x + 11 = 18 \)
   - \( 3x^2 + 12x - 7 = 0 \)
   - \( x = \frac{-12 \pm \sqrt{144 - 4 \cdot 3 \cdot (-7)}}{6} \)
   - \( x = \frac{-12 \pm \sqrt{144 + 84}}{6} \)
   - \( x = \frac{-12 \pm 12}{6} \)
   - The solution set is \( \{0, -2\} \).

5. \( x^2 + 3x + 2 = 0 \)  
   - \( x^2 + 3x + 2 = 0 \)
   - \( x = \frac{-3 \pm \sqrt{9 - 8}}{2} \)
   - \( x = \frac{-3 \pm 1}{2} \)
   - The solution set is \( \{-2, -1\} \).

6. \( 2x^2 + 8x + 7 = 0 \)  
   - \( 2x^2 + 8x + 7 = 0 \)
   - \( x = \frac{-8 \pm \sqrt{64 - 56}}{4} \)
   - \( x = \frac{-8 \pm 2}{4} \)
   - The solution set is \( \{-3, -1\} \).

7. \( x^2 - 7x + 12 = 0 \)  
   - \( x^2 - 7x + 12 = 0 \)
   - \( x = \frac{7 \pm \sqrt{49 - 48}}{2} \)
   - \( x = \frac{7 \pm 1}{2} \)
   - The solution set is \( \{2, 5\} \).

8. \( x^2 + 4x + 3 = 0 \)  
   - \( x^2 + 4x + 3 = 0 \)
   - \( x = \frac{-4 \pm \sqrt{16 - 12}}{2} \)
   - \( x = \frac{-4 \pm 2}{2} \)
   - The solution set is \( \{-1, -3\} \).

9. \( x^2 + 7x + 10 = 0 \)  
   - \( x^2 + 7x + 10 = 0 \)
   - \( x = \frac{-7 \pm \sqrt{49 - 40}}{2} \)
   - \( x = \frac{-7 \pm 3}{2} \)
   - The solution set is \( \{-5, -2\} \).

10. \( x^2 - 10x + 16 = 0 \)  
    - \( x^2 - 10x + 16 = 0 \)
    - \( x = \frac{10 \pm \sqrt{100 - 64}}{2} \)
    - \( x = \frac{10 \pm 6}{2} \)
    - The solution set is \( \{4, 6\} \).

**Example 1**

Find the value of \( c \) that makes \( x^2 + 22x + c \) a perfect square trinomial.

Then write the trinomial as the square of a binomial.

- **Step 1** 
  - \( b = 22; \frac{b}{2} = 11 \)
  - \( c = 11^2 = 121 \)
  - The trinomial is \( x^2 + 22x + 121 \), which can be written as \( (x + 11)^2 \).

- **Step 2**
  - \( a = 1; b = -22; c = 121 \)
  - \( \frac{-b}{2a} = \frac{22}{2} = 11 \)
  - The solution set is \( \{-22 \pm 11\} \).

**Example 2**

Solve \( 2x^2 - 8x - 4 = 0 \) by completing the square.

- **Step 1**
  - \( 2x^2 - 8x = 4 \)
  - Divide each side by 2.
  - \( x^2 - 4x = 2 \)
  - \( x^2 - 4x + 4 = 2 + 4 \)
  - \( (x - 2)^2 = 6 \)

**Exercises**

Find the value of \( c \) that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

1. \( x^2 + 10x + c = 25; (x - 5)^2 \)
2. \( x^2 + 60x + c = 900; (x + 30)^2 \)
3. \( x^2 - 3x + c = \frac{9}{4}; (x - \frac{3}{2})^2 \)
4. \( x^2 + 3.2x + c = \frac{5}{16}; (x + \frac{1}{8})^2 \)
5. \( x^2 + 2.5x + c = 1.5625; (x - 0.5)^2 \)

Solve each equation by completing the square.

1. \( x^2 - 4y = 5 = 0 \)
2. \( x^2 - 8y - 65 = 0 \)
3. \( x^2 - 10y + 21 = 0 \)
4. \( x^2 - 3x + 1 = 0 \)
5. \( x^2 + 13x - 7 = 0 \)
6. \( x^2 + 40x - 9 = 0 \)
7. \( x^2 + 4x + 1 = 0 \)
8. \( x^2 + 12y + 4 = 0 \)
9. \( x^2 + 3x - 8 = 0 \)

- **Step 1**
  - \( x = \frac{-b}{2a} \)
  - \( x = \frac{-b}{2a} \)
  - \( x = \frac{-3}{2} \)
  - \( x = -\frac{3}{2} \)
  - \( x = -3.75 \)
  - \( x = -3.75 \)
  - \( x = -3.75 \)

- **Step 2**
  - \( \pm \sqrt{a} \)
  - \( \pm \sqrt{a} \)
  - \( \pm \sqrt{a} \)
  - \( \pm \sqrt{a} \)
  - \( \pm \sqrt{a} \)
  - \( \pm \sqrt{a} \)
  - \( \pm \sqrt{a} \)
  - \( \pm \sqrt{a} \)
  - \( \pm \sqrt{a} \)

- **Step 3**
  - \( a \) \( b \) \( c \)
  - \( a \) \( b \) \( c \)
  - \( a \) \( b \) \( c \)
  - \( a \) \( b \) \( c \)
  - \( a \) \( b \) \( c \)
  - \( a \) \( b \) \( c \)
  - \( a \) \( b \) \( c \)
  - \( a \) \( b \) \( c \)
  - \( a \) \( b \) \( c \)

- **Step 4**
  - \( a \) \( b \) \( c \)
  - \( a \) \( b \) \( c \)
  - \( a \) \( b \) \( c \)
  - \( a \) \( b \) \( c \)
  - \( a \) \( b \) \( c \)
  - \( a \) \( b \) \( c \)
  - \( a \) \( b \) \( c \)
  - \( a \) \( b \) \( c \)
  - \( a \) \( b \) \( c \)
4-5 Skills Practice

Completing the Square

Solve each equation by completing the Square. Round to the nearest hundredth if necessary.

1. \(x^2 - 8x + 16 = 1\) \(3, 5\)
2. \(x^2 + 4x + 4 = 1\) \(-1, -3\)
3. \(x^2 + 12x + 36 = 25\) \(-1, -11\)
4. \(4x^2 - 4x + 1 = 9\) \(-1, 2\)
5. \(x^2 + 4x + 4 = 2\) \(-3.41, -0.59\)
6. \(x^2 - 2x + 1 = 5\) \(-1.24, 3.24\)
7. \(x^2 - 6x + 9 = 7\) \(0.35, 5.65\)
8. \(x^2 + 16x + 64 = 15\) \(-11.87, -4.13\)

Find the value of \(c\) that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

9. \(x^2 + 10x + c\) \(25; (x + 5)^2\)
10. \(x^2 - 14x + c\) \(49; (x - 7)^2\)
11. \(x^2 + 24x + c\) \(144; (x + 12)^2\)
12. \(x^2 + 5x + c\) \(\frac{25}{4}; \left(x + \frac{5}{2}\right)^2\)
13. \(x^2 - 9x + c\) \(\frac{81}{4}; \left(x - \frac{9}{2}\right)^2\)
14. \(x^2 - x + c\) \(\frac{1}{4}; \left(x - \frac{1}{2}\right)^2\)

Solve each equation by completing the square.

15. \(x^2 - 13x + 36 = 0\) \(4, 9\)
16. \(x^2 + 3x = 0\) \(0, -3\)
17. \(x^2 + x - 6 = 0\) \(2, -3\)
18. \(x^2 - 4x = -13 = 0\) \(2 \pm \sqrt{17}\)
19. \(2x^2 + 7x - 4 = 0\) \(-4, \frac{1}{2}\)
20. \(3x^2 + 2x - 1 = 0\) \(\frac{1}{3}, -1\)
21. \(x^2 + 3x - 6 = 0\) \(-3 \pm \sqrt{33}\)
22. \(x^2 - x - 3 = 0\) \(1 \pm \sqrt{13}\)
23. \(x^2 = -11 \pm i\sqrt{11}\)
24. \(x^2 - 2x + 4 = 0\) \(1 \pm i\sqrt{3}\)
25. \(2x^2 + 4x - 3 = 0\)
26. \(x^2 + x + 3 = 0\)
27. \(2x^2 + 10x + 5 = 0\)
28. \(3x^2 + 3x + 6 = 0\)
29. \(2x^2 + 5x + 6 = 0\)
30. \(7x^2 + 6x + 2 = 0\)

31. GEOMETRY When the dimensions of a cube are reduced by 4 inches on each side, the surface area of the new cube is 864 square inches. What were the dimensions of the original cube? 16 in. by 16 in. by 16 in.

32. INVESTMENTS The amount of money \(A\) in an account in which \(P\) dollars are invested for 2 years is given by the formula \(A = P(1 + r)^2\), where \(r\) is the interest rate compounded annually. If an investment of $800 in the account grows to $882 in two years, at what interest rate was it invested? 5%
4-5 Word Problem Practice

Completing the Square

1. COMPLETING THE SQUARE
Samanta needs to solve the equation $x^2 - 12x = 40$.
What must she do to each side of the equation to complete the square? Add 36.

2. ART
The area in square inches of the drawing Foliage by Paul Cézanne is approximated by the equation $y = x^2 - 40x + 396$. Complete the square and find the two roots, which are equal to the approximate length and width of the drawing.

18 inches by 22 inches

3. COMPOUND INTEREST
Nikki invested $1000 in a savings account with interest compounded annually. After two years the balance in the account is $1210. Use the compound interest formula $A = P(1 + r)^t$ to find the annual interest rate. 10%

4. REACTION TIME
Lauren was eating lunch when she saw her friend Jason approach. The room was crowded and Jason had to lift his tray to avoid obstacles. Suddenly, a glass on Jason’s lunch tray tipped and fell off the tray. Lauren lunged forward and managed to catch the glass just before it hit the ground. The height $h$, in feet, of the glass $t$ seconds after it was dropped is given by $h = -16t^2 + 4.5$. Lauren caught the glass when it was six inches off the ground. How long was the glass in the air before Lauren caught it? 0.5 second

5. PARABOLAS
A parabola is modeled by $y = x^2 - 10x + 28$. Jane’s homework problem requires that she find the vertex of the parabola. She uses the completing square method to express the function in the form $y = (x - h)^2 + k$, where $h, k$ is the vertex of the parabola. Write the function in the form used by Jane.

$y = (x - 5)^2 + 3$

6. AUDITORIUM SEATING
The seats in an auditorium are arranged in a square grid pattern. There are 45 rows and 45 columns of chairs. For a special concert, organizers decide to increase seating by adding $n$ rows and $n$ columns to make a square pattern of seating $45 + n$ seats on a side.

a. How many seats are there after the expansion?

$n^2 + 90n + 2025$

b. What is $n$ if organizers wish to add 1000 seats?

10

c. If organizers do add 1000 seats, what is the seating capacity of the auditorium?

3025

4-5 Enrichment

The Golden Quadratic Equations

A golden rectangle has the property that its length can be written as $a + b$, where $a$ is the width of the rectangle and $\frac{a + b}{a} = \frac{\sqrt{5} + 1}{2}$. Any golden rectangle can be divided into a square and a smaller golden rectangle, as shown.

The proportion used to define golden rectangles can be used to derive two quadratic equations. These are sometimes called golden quadratic equations.

Solve each problem.

1. In the proportion for the golden rectangle, let $a$ equal 1. Write the resulting quadratic equation and solve for $b$.

$b^2 + b - 1 = 0$

$b = -1 + \sqrt{5}$

2. In the proportion, let $b$ equal 1. Write the resulting quadratic equation and solve for $a$.

$a^2 - a - 1 = 0$

$a = \frac{1 + \sqrt{5}}{2}$

3. Describe the difference between the two golden quadratic equations you found in exercises 1 and 2. The signs of the first-degree terms are opposite.

4. Show that the positive solutions of the two equations in exercises 1 and 2 are reciprocals.

$$\left(1 - \sqrt{5}\right) \left(1 + \sqrt{5}\right) = \frac{-1 + \sqrt{5}}{4} = -\frac{1}{4} + \frac{5}{4} = 1$$

5. Use the Pythagorean Theorem to find a radical expression for the diagonal of a golden rectangle when $a = 1$.

$$d = \frac{\sqrt{10 - 2\sqrt{5}}}{2}$$

6. Find a radical expression for the diagonal of a golden rectangle when $b = 1$.

$$d = \frac{\sqrt{10 + 2\sqrt{5}}}{2}$$
### 4-6 Study Guide and Intervention

#### The Quadratic Formula and the Discriminant

**Quadratic Formula** The Quadratic Formula can be used to solve any quadratic equation once it is written in the form $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Example** Solve $x^2 - 5x = 14$ by using the Quadratic Formula.

Rewrite the equation as $x^2 - 5x - 14 = 0$.

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-14)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{57}}{2}$$

The solutions are $2$ and $7$.

### Exercises

Solve each equation by using the Quadratic Formula.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2 + 2x - 35 = 0$</td>
<td>$5, -7$</td>
</tr>
<tr>
<td>$2x^2 + 10x + 24 = 0$</td>
<td>$3, 8$</td>
</tr>
<tr>
<td>$3x^2 - 11x + 24 = 0$</td>
<td>$-$</td>
</tr>
<tr>
<td>$4x^2 + 19x - 5 = 0$</td>
<td>$5, 14$</td>
</tr>
<tr>
<td>$5x^2 + 9x + 1 = 0$</td>
<td>$1, 2$</td>
</tr>
<tr>
<td>$6x^2 - x - 15 = 0$</td>
<td>$3, -5$</td>
</tr>
<tr>
<td>$7x^2 + 5x = 2$</td>
<td>$2, -3$</td>
</tr>
<tr>
<td>$8x^2 - y = 15 = 0$</td>
<td>$4, 3$</td>
</tr>
<tr>
<td>$9x^2 - 16x + 16 = 0$</td>
<td>$2, -2$</td>
</tr>
<tr>
<td>$10x^2 - 3x - 2 = 0$</td>
<td>$3, 2$</td>
</tr>
<tr>
<td>$11x^2 - 7x - 50 = 25 = 0$</td>
<td>$5, 5\sqrt{3}$</td>
</tr>
<tr>
<td>$12x^2 - 10x - 50 = 0$</td>
<td>$3, 2$</td>
</tr>
<tr>
<td>$13x^2 + 6x - 23 = 0$</td>
<td>$-3 \pm 4\sqrt{2}$</td>
</tr>
<tr>
<td>$14x^2 - 12x - 63 = 0$</td>
<td>$3 \pm 6\sqrt{2}$</td>
</tr>
<tr>
<td>$15x^2 - 6x + 21 = 0$</td>
<td>$3 \pm 2i\sqrt{3}$</td>
</tr>
</tbody>
</table>

### Roots and the Discriminant

The expression under the radical sign, $b^2 - 4ac$, in the Quadratic Formula is called the discriminant.

<table>
<thead>
<tr>
<th>Discriminant</th>
<th>Type and Number of Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b^2 - 4ac &gt; 0$ and a perfect square</td>
<td>2 rational roots</td>
</tr>
<tr>
<td>$b^2 - 4ac = 0$</td>
<td>1 rational root</td>
</tr>
<tr>
<td>$b^2 - 4ac &lt; 0$</td>
<td>2 complex roots</td>
</tr>
</tbody>
</table>

**Example** Find the value of the discriminant for each equation. Then describe the number and type of roots for the equation.

**a.** $2x^2 + 5x + 3$

- The discriminant is $b^2 - 4ac = (5)^2 - 4(2)(3) = 1$. The discriminant is a perfect square, so the equation has 2 rational roots.

**b.** $3x^2 - 2x + 5$

- The discriminant is $b^2 - 4ac = (-2)^2 - 4(3)(5) = -56$. The discriminant is negative, so the equation has 2 complex roots.

### Exercises (Lesson 4.6)

Complete parts a–e for each quadratic equation.

**a.** Find the value of the discriminant.
**b.** Describe the number and type of roots.
**c.** Find the exact solutions by using the Quadratic Formula.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^2 + 12p = -4$</td>
<td>$2$ irrational roots; $1$ rational root; $\frac{1}{2}$</td>
</tr>
<tr>
<td>$2x^2 - 6x + 1 = 0$</td>
<td>$2$ irrational roots; $1$ rational root; $\frac{1}{2}$</td>
</tr>
<tr>
<td>$3x^2 - 7x = 4 = 0$</td>
<td>$2$ irrational roots; $1$ rational root; $\frac{1}{2}$</td>
</tr>
<tr>
<td>$4x^2 + 4x - 4 = 0$</td>
<td>$2$ irrational roots; $1$ rational root; $\frac{1}{2}$</td>
</tr>
<tr>
<td>$5x^2 - 36x + 7 = 0$</td>
<td>$2$ irrational roots; $1$ rational root; $\frac{1}{2}$</td>
</tr>
<tr>
<td>$6x^2 - 4x + 11 = 0$</td>
<td>$2$ irrational roots; $1$ rational root; $\frac{1}{2}$</td>
</tr>
<tr>
<td>$2 \pm 2\sqrt{2}$</td>
<td>$1 \pm \sqrt{10}$</td>
</tr>
<tr>
<td>$7x^2 - 7x + 6 = 0$</td>
<td>$2$ irrational roots; $1$ rational root; $\frac{1}{2}$</td>
</tr>
<tr>
<td>$8x^2 - 8m = -14$</td>
<td>$2$ irrational roots; $1$ rational root; $\frac{1}{2}$</td>
</tr>
<tr>
<td>$9.25x^2 - 40x = -16$</td>
<td>$2$ irrational roots; $1$ rational root; $\frac{1}{2}$</td>
</tr>
<tr>
<td>$3$ irrational roots; $2$ irrational roots; $1$ rational root; $\frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td>$10.42x^2 + 29 = 0$</td>
<td>$2$ complex roots; $2$ irrational roots; $1$ rational root; $\frac{1}{2}$</td>
</tr>
<tr>
<td>$11.6x^2 + 26x + 8 = 0$</td>
<td>$2$ complex roots; $2$ irrational roots; $1$ rational root; $\frac{1}{2}$</td>
</tr>
<tr>
<td>$12.4x^2 - 4x = 11 = 0$</td>
<td>$2$ complex roots; $2$ irrational roots; $1$ rational root; $\frac{1}{2}$</td>
</tr>
</tbody>
</table>
Chapter 4

4-6 Skills Practice

The Quadratic Formula and the Discriminant

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.
b. Describe the number and type of roots.
c. Find the exact solutions by using the Quadratic Formula.

1. \( x^2 - 8x + 16 = 0 \)
   - 0; 1 rational root; 4

2. \( x^2 - 11x - 26 = 0 \)
   - 225; 2 rational roots; -2, 13

3. \( 3x^2 - 2x = 0 \)
   - 4; 2 rational roots; 0, \( \frac{2}{3} \)

4. \( 20x^2 + 7x - 3 = 0 \)
   - 289; 2 rational roots; \(- \frac{3}{5}, \frac{1}{4} \)

5. \( 5x^2 - 6 = 0 \)
   - 6; \( x^2 = \frac{6}{5} \)

6. \( 120; 2 \) irrational roots; \( \pm \frac{\sqrt{30}}{5} \)
   - 24; 2 irrational roots; \( \pm \sqrt{6} \)

7. \( x^2 + 8x + 13 = 0 \)
   - 8.5; \( x^2 - x = 1 = 0 \)
   - 21; 2 irrational roots; \( 1 \pm \frac{\sqrt{21}}{10} \)

8. \( x^2 - 2x - 17 = 0 \)
   - 10; \( x^2 + 49 = 0 \)

9. \( x^2 - 2x + 1 = 0 \)
   - 12; \( 3x^2 - 3x = -2 \)
   - 2 complex roots; \( 1 \pm \sqrt{3} \)

10. \( x^2 - x + 1 = 0 \)
    - \( -3; 2 \) complex roots; \( \frac{1 \pm \sqrt{3}}{2} \)

11. \( x^2 - 5x + 6 = 0 \)
    - 2 rational roots; \( 2, 3 \)

12. \( x^2 + 3x + 2 = 0 \)
    - \( 2 \) complex roots; \( -1, -2 \)

13. \( x^2 - 3x + 2 = 0 \)
    - \( 2 \) complex roots; \( 1, 2 \)

14. \( x^2 - 15 = 0 \)
    - \( 2 \) rational roots; \( \pm \sqrt{15} \)

15. \( x^2 - 16x + 64 = 0 \)
    - 4; \( x^2 - 8x + 16 = 0 \)

16. \( x^2 = 3x \)
    - 12; \( 2 \) irrational roots; \( 1 \pm \frac{\sqrt{3}}{2} \)

17. \( x^2 - 2x + 4 = 0 \)
    - 2 complex roots; \( -1 \pm \sqrt{3} \)

18. \( x^2 = 3x + 2 = 0 \)
    - \( 2 \) irrational roots; \( 1 \pm \frac{\sqrt{3}}{2} \)

19. \( x^2 = 2x + 4 = 0 \)
    - 2 complex roots; \( 1 \pm \frac{\sqrt{3}}{2} \)

20. \( x^2 = 3x + 2 = 0 \)
    - \( 2 \) irrational roots; \( 1 \pm \frac{\sqrt{3}}{2} \)

21. \( x^2 = 2x + 4 = 0 \)
    - \( 2 \) irrational roots; \( 1 \pm \frac{\sqrt{3}}{2} \)

22. \( x^2 - 7x + 4 = 0 \)
    - \( 2 \) irrational roots; \( 1 \pm \frac{\sqrt{3}}{2} \)

23. \( x^2 + 1 = 4x \)
    - \( 1 \pm \frac{1}{2} \)

24. \( x^2 + 2x + 3 = 0 \)
    - \( -1 \pm \frac{\sqrt{4}}{2} \)

25. PARACHUTING Ignoring wind resistance, the distance \( d(t) \) in feet that a parachutist falls in \( t \) seconds can be estimated using the formula \( d(t) = 16t^2 \). If a parachutist jumps from an airplane and falls for 1100 feet before opening her parachute, how many seconds pass before she opens the parachute? about 8.3 s

26. GRAVITATION The height \( h(t) \) in feet of an object \( t \) seconds after it is propelled straight up from the ground with an initial velocity of 60 feet per second is modeled by the equation \( h(t) = -16t^2 + 60t \). At what times will the object be at a height of 56 feet? 1.75 s, 2 s

27. STOPPING DISTANCE The formula \( d = 0.05v^2 + 1.1v \) estimates the minimum stopping distance \( d \) in feet for a car traveling \( v \) miles per hour. If a car stops in 200 feet, what is the fastest it could have been traveling when the driver applied the brake? about 53.2 mi/h
Word Problem Practice

The Quadratic Formula and the Discriminant

1. PARABOLAS The graph of a quadratic equation of the form \( y = ax^2 + bx + c \) is shown below.

Is the discriminant \( b^2 - 4ac \) positive, negative, or zero? Explain.
Negative; the equation has no real solutions so the discriminant is negative.

2. TANGENT Kathleen is trying to find \( b \) so that the \( x \)-axis is tangent to the parabola \( y = x^2 + bx + 4 \). She finds one value that works, \( b = 4 \). Is this the only value that works? Explain.
No, \( b = -4 \) also works; the \( x \)-axis will be tangent when the discriminant \( b^2 - 16 \) is zero. This happens when \( b = 4 \) or \(-4\).

3. SPORTS In 1990, American Randy Barnes set the world record for the shot put. His throw can be described by the equation \( y = -16x^2 + 360x \). Use the Quadratic Formula to find how far his throw was to the nearest foot.
23 ft

4. EXAMPLES Give an example of a quadratic function \( f(x) \) that has the following properties.

I. The discriminant of \( f \) is zero.
II. There is no real solution of the equation \( f(x) = 0 \).
Sample answer: \( f(x) = x^2 \)

5. TANGENTS The graph of \( y = x^2 \) is a parabola that passes through the point \( (1, 1) \). The line \( y = mx - m + 1 \), where \( m \) is a constant, also passes through the point \( (1, 1) \).

a. To find the points of intersection between the line \( y = mx - m + 1 \) and the parabola \( y = x^2 \), set \( x^2 = mx - m + 1 \) and then solve for \( x \).
Rearranging terms, this equation becomes \( x^2 - mx - m + 1 = 0 \). What is the discriminant of this equation?
\( x^2 - 4m + 4 \)

b. For what value of \( m \) is there only one point of intersection? Explain the meaning of this in terms of the corresponding line and the parabola.
\( m = 2 \); the parabola \( y = x^2 \) and the line \( y = 2x - 1 \) have exactly one point of intersection at \((1, 1)\). In other words, this line is tangent to the parabola at \((1, 1)\).

Enrichment

Sum and Product of Roots

Sometimes you may know the roots of a quadratic equation without knowing the equation itself. Using your knowledge of factoring to solve an equation, you can work backward to find the quadratic equation. The rule for finding the sum and product of roots is as follows:

- If the roots of \( ax^2 + bx + c = 0 \), with \( a \neq 0 \), are \( x_1 \) and \( x_2 \), then \( x_1 + x_2 = -\frac{b}{a} \) and \( x_1 \cdot x_2 = \frac{c}{a} \)

Example

Write a quadratic equation that has the roots 3 and \(-8\).

The roots are \( 3 \) and \(-8\).

3 + \(-8\) = \(-5\) Add the roots.

\( (3)(-8) = -24 \) Multiply the roots.

Equation: \( x^2 + 5x - 24 = 0 \)

Exercises

Write a quadratic equation that has the given roots.

1. \( 6, -9 \)
2. \( 5, -1 \)
3. \( 6, 6 \)
4. \( 4 \pm \sqrt{3} \)
5. \(-\frac{3}{2}, \frac{9}{2} \)
6. \(-2 \pm 3\sqrt{5} \)

Find \( k \) such that the number given is a root of the equation.

7. \( 7; 2x^2 + kx - 21 = 0 \)
8. \(-2; x^2 - 13x + k = 0 \)

\(-11 \quad -30 \)
4-6 Spreadsheet Activity

Approximating the Real Zeros of Polynomials

You have learned the Location Principle, which can be used to approximate the real zeros of a polynomial.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>4.5</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

In the spreadsheet above, the positive real zero of \( f(x) = x^2 - 2 \) can be approximated in the following way. Set the spreadsheet preference to manual calculation. The values in A2 and B2 are the endpoints of a range of values. The values in D2 through J2 are values equally in the interval from A2 to B2. The formulas for these values are \( A2 + (B2 - A2)/6, A2 + 2*(B2 - A2)/6, A2 + 3*(B2 - A2)/6, A2 + 4*(B2 - A2)/6, A2 + 5*(B2 - A2)/6, \\) and B2, respectively.

Row 3 gives the function values at these points. The function \( f(x) = x^2 - 2 \) is entered into the spreadsheet in Cell D3 as \( D2^2 - 2 \). This function is then copied to the remaining cells in the row.

You can use this spreadsheet to study the function values at the points in cells D2 through J2. The value in cell D3 is positive and the value in cell G3 is negative, so there must be a zero between \(-1.6867 \) and \( 0 \). Enter these values in cells A2 and B2, respectively, and recalculate the spreadsheet. (You will have to recalculate a number of times.) The result is a new table from which you can see that there is a zero between \( 1.4141 \) and \( 1.4143 \). Because these values agree to three decimal places, the zero is about \( 1.414 \). This can be verified by using algebra.

By solving \( x^2 - 2 = 0 \), we obtain \( x = \pm \sqrt{2} \). The positive root is \( x = \frac{\pm \sqrt{2}}{2} \). Which verifies the result.

Exercises

1. Use a spreadsheet like the one above to approximate the zero of \( f(x) = 3x - 2 \) to three decimal places. Then verify your answer by using algebra to find the exact value of the root. The spreadsheet gives \( x = 0.667 \). By solving for \( x \) algebraically, \( x = 2/3 \). So, the approximation is correct.

2. Use a spreadsheet like the one above to approximate the real zeros of \( f(x) = x^2 + 2x + 0.5 \). Round your answer to four decimal places. Then verify your answer by using the quadratic formula. The process gives \(-1.7071 \) and \(-0.2929 \) to the nearest ten-thousandth. The quadratic formula gives \( x = -1 \pm \frac{-0.2929 \approx -1.7071 \text{ and } -1 + \sqrt{2} \approx -0.2929.} \)

3. Use a spreadsheet like the one above to approximate the real zero of \( f(x) = x^3 - \frac{3}{2}x^2 - 6x - 2 \) between \(-0.4 \) and \(-0.3 \). \(-0.3781 \) to the nearest ten-thousandth.

4-7 Study Guide and Intervention

Transformations of Quadratic Graphs

Write Quadratic Equations in Vertex Form

A quadratic function is easier to graph when it is in vertex form. You can write a quadratic function of the form \( y = ax^2 + bx + c \) in vertex form by completing the square.

Example

Write \( y = 2x^2 - 12x + 25 \) in vertex form. Then graph the function.

\[ y = 2x^2 - 12x + 25 \]
\[ y = 2x^2 - 6x + 9 + 25 - 18 \]
\[ y = 2(x - 3)^2 + 7 \]

The vertex form of the equation is \( y = 2(x - 3)^2 + 7 \).

Exercises

Write each equation in vertex form. Then graph the function.

1. \( y = x^2 - 10x + 32 \)
2. \( y = x^2 + 6x \)
3. \( y = x^2 - 8x + 6 \)
4. \( y = -4x^2 + 16x - 11 \)
5. \( y = 3x^2 - 12x + 5 \)
6. \( y = 5x^2 - 10x + 9 \)
Transformations of Quadratic Graphs

Parabolas can be transformed by changing the values of the constants $a$, $h$, and $k$ in the vertex form of a quadratic equation: $y = a(x - h)^2 + k$.

- The sign of $a$ determines whether the graph opens upward ($a > 0$) or downward ($a < 0$).
- The absolute value of $a$ also causes a dilation (enlargement or reduction) of the parabola. The parabola becomes narrower if $|a| > 1$ and wider if $|a| < 1$.
- The value of $h$ translates the parabola horizontally. Positive values of $h$ slide the graph to the right and negative values slide the graph to the left.
- The value of $k$ translates the graph vertically. Positive values of $k$ slide the graph upward and negative values slide the graph downward.

Example: Graph $y = (x + 7)^2 + 3$.

- Rewrite the equation as $y = (x - (-7))^2 + 3$.
- Because $h = -7$ and $k = 3$, the vertex is at $(-7, 3)$. The axis of symmetry is $x = -7$. Because $a = 1$, we know that the graph opens up, and the graph is the same width as the graph of $y = x^2$.
- Translate the graph of $y = x^2$ seven units to the left and three units up.

Exercises

Graph each function.

1. $y = -2x^2 + 2$
2. $y = -3x - 1^2$
3. $y = 2x + 2^3 + 3$

Transformations of Quadratic Graphs

Write each quadratic function in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.

<table>
<thead>
<tr>
<th>$y = a(x - h)^2 + k$</th>
<th>$y = -(x - h)^2 + k$</th>
<th>$y = -(x - h)^2 - k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0, 2); x = 2$; up</td>
<td>$(0, 4); x = 0$; down</td>
<td>$(0, -6); x = 0$; up</td>
</tr>
<tr>
<td>$y = -3(x + 2)^2$</td>
<td>$y = -5(x - 0)^2 + 9$</td>
<td>$y = (x - 2)^2 - 18$</td>
</tr>
<tr>
<td>$(-5, 0); x = -5$; down</td>
<td>$(0, 9); x = 0$; down</td>
<td>$(2, -18); x = 2$; up</td>
</tr>
<tr>
<td>$y = x^2 - 2x - 5$</td>
<td>$y = x^2 + 6x + 2$</td>
<td>$y = -3x^2 + 24x$</td>
</tr>
<tr>
<td>$(1, -6); x = 1$; up</td>
<td>$y = (x + 3)^2 - 7$</td>
<td>$y = -3(x - 4)^2 + 48$</td>
</tr>
<tr>
<td>$(-3, -7); x = -3$; up</td>
<td>$(4, 48); x = 4$; down</td>
<td></td>
</tr>
</tbody>
</table>

Graph each function.

10. $y = (x - 3)^2 - 1$
11. $y = (x + 1)^2 + 2$
12. $y = -(x - 4)^2 - 4$

13. $y = -\frac{1}{2}(x + 2)^2$
14. $y = -3x^2 + 4$
15. $y = x^2 + 6x + 4$
4-7 Practice
Transformations of Quadratic Graphs
Write each equation in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.

1. \(y = -6x^2 - 24x - 25\)  
2. \(y = 2x^2 + 2\)  
3. \(y = -4x^2 + 8x\)

\(-2, -1\); \(x = -2, \text{down}\)  
\((0, 2); \ x = 0, \text{up}\)  
\((1, 4); x = 1, \text{down}\)

4. \(y = x^2 + 10x + 20\)  
5. \(y = 2x^2 + 12x + 18\)  
6. \(y = 3x^2 - 6x + 5\)

\((0, 2); \ x = 0, \text{up}\)  
\((1, 2); x = 1, \text{up}\)

7. \(y = -2x^2 - 16x - 32\)  
8. \(y = -3x^2 + 18x - 21\)  
9. \(y = 2x^2 + 16x + 29\)

\((-4, 0); \ x = -4, \text{down}\)  
\((3, 5); x = 3, \text{down}\)  
\((-4, -3); x = -4, \text{up}\)

Graph each function.

10. \(y = (x + 3)^2 - 1\)  
11. \(y = -x^2 + 6x - 5\)  
12. \(y = 2x^2 - 2x + 1\)

13. Write an equation for a parabola with vertex at \((1, 3)\) that passes through \((-2, -15)\).

\(y = -2(x - 1)^2 + 3\)

14. Write an equation for a parabola with vertex at \((-3, 0)\) that passes through \((3, 18)\).

\(y = \frac{1}{2}(x + 3)^2\)

15. Baseball. The height \(h\) of a baseball \(t\) seconds after being hit is given by \(h = -16t^2 + 80t + 3\). What is the maximum height that the baseball reaches, and when does this occur?

103 ft; 2.5 s

16. Sculpture. A modern sculpture in a park contains a parabolic arch that starts at the ground and reaches a maximum height of 10 feet after a horizontal distance of 4 feet. Write a quadratic function in vertex form that describes the shape of the outside of the arch, where \(y\) is the height of a point on the arch and \(x\) is its horizontal distance from the left-hand starting point of the arch.

\(y = -\frac{5}{8}(x - 4)^2 + 10\)

4-7 Word Problem Practice
Transformations of Quadratic Graphs
1. Arches. A parabolic arch is used as a bridge support. The graph of the arch is shown below.

If the equation that corresponds to this graph is written in the form \(y = a(x - h)^2 + k\), what are \(a\) and \(k\)?

\(a = -1\) and \(k = 5\)

2. Translations. For a computer animation, Barbara uses the quadratic function \(f(x) = -42x^2 - 207 + 16000\) to help her simulate an object tossed on another planet. For one skit, she had to use the function \(f(x) + 50 - 8000\) instead of \(f(x)\). Where is the vertex of the graph of \(y = f(x) + 50 - 8000\)?

\((15, 8800)\)

3. Bridges. The shape formed by the main cables of the Golden Gate Bridge approximately follows the equation \(y = 0.0002x^2 - 0.23x + 227\). Graph the parabola formed by one of the cables.

4. Water Jets. The graph shows the path of a jet of water.

The equation corresponding to this graph is \(y = a(x - h)^2 + k\). What are \(a\), \(h\), and \(k\)?

\(a = -2, \ h = 2, k = 6\)

5. Profit. A theater operator predicts that the theater can make \(-4x^2 + 160x\) dollars per show if tickets are priced at \(x\) dollars.

a. Rewrite the equation \(y = -4x^2 + 160x\) in the form \(y = ax^2 + bx + c\).

\(y = -4(x - 20)^2 + 1600\)

b. What is the vertex of the parabola and what is its axis of symmetry?

vertex at \((20, 1600)\); axis is \(x = 20\)

c. Graph the parabola.
4-7 Enrichment

A Shortcut to Complex Roots

When graphing a quadratic function, the real roots are shown in the graph. You have learned that quadratic functions can also have imaginary roots that cannot be seen on the graph of the function. However, there is a way to graphically represent the complex roots of a quadratic function.

**Example**

Find the complex roots of the quadratic function \(y = x^2 - 4x + 5\).

**Step 1** Graph the function.

![Graph of quadratic function](image)

**Step 2** Reflect the graph over the horizontal line containing the vertex. In this example, the vertex is (2, 1).

![Reflected graph](image)

**Step 3** The real part of the complex root is the point halfway between the \(x\)-intercepts of the reflected graph and the imaginary part of the complex roots are + and – half the distance between the \(x\)-intercepts of the reflected graph. So, in this example, the complex roots are 2 + 1i and 2 – 1i.

**Exercises**

Using this method, find the complex roots of the following quadratic functions.

1. \(y = x^2 + 2x + 5\) 
   -1 + 2i, -1 - 2i
2. \(y = x^2 + 4x + 8\) 
   -2 + 2i, -2 - 2i
3. \(y = x^2 + 6x + 13\) 
   -3 + 2i, -3 - 2i
4. \(y = x^2 + 2x + 17\) 
   -1 + 4i, -1 - 4i

4-8 Study Guide and Intervention

Quadratic Inequalities

**Graph Quadratic Inequalities**

To graph a quadratic inequality in two variables, use the following steps:

1. Graph the related quadratic equation, \(y = ax^2 + bx + c\).
   Use a dashed line for < or >; use a solid line for \(\leq\) or \(\geq\).

2. Test a point inside the parabola.
   If it satisfies the inequality, shade the region inside the parabola; otherwise, shade the region outside the parabola.

**Example**

Graph the inequality \(y > x^2 + 6x + 7\).

First graph the equation \(y = x^2 + 6x + 7\). By completing the square, you get the vertex form of the equation \(y = (x + 3)^2 - 2\), so the vertex is (-3, -2). Make a table of values around \(x = -3\), and graph. Since the inequality includes >, use a dashed line.

Test the point (-3, 0), which is inside the parabola. Since \((-3)^2 + 6(-3) + 7 = -2\) and \(0 > -2\), (-3, 0) satisfies the inequality. Therefore, shade the region inside the parabola.

**Exercises**

Graph each inequality.

1. \(y > x^2 - 8x + 17\)
2. \(y \leq x^2 + 6x + 4\)
3. \(y \geq x^2 + 2x + 2\)
4. \(y < -x^2 + 4x - 6\)
5. \(y \geq 2x^2 + 4x\)
6. \(y > -2x^2 - 4x + 2\)
Chapter 4

4-8 Study Guide and Intervention (continued)

**Quadratic Inequalities**

Quadratic inequalities in one variable can be solved graphically or algebraically.

### Graphical Method
To solve $ax^2 + bx + c < 0$.
1. First graph $y = ax^2 + bx + c$. The solution consists of the $x$-values for which the graph is below the $x$-axis.
2. To solve $ax^2 + bx + c > 0$.
   - First graph $y = ax^2 + bx + c$. The solution consists of the $x$-values for which the graph is above the $x$-axis.

### Algebraic Method
Find the roots of the related quadratic equation by factoring, completing the square, or using the Quadratic Formula.
- Two roots divide the number line into 3 intervals.
- Test a value in each interval to see which intervals are solutions.

If the inequality involves $\leq$ or $\geq$, the roots of the related equation are included in the solution set.

**Example** Solve the inequality $x^2 - x - 6 \leq 0$.
First find the roots of the related equation $x^2 - x - 6 = 0$. The equation factors as $(x - 3)(x + 2) = 0$, so the roots are 3 and -2.
The graph opens up with $x$-intercepts 3 and -2, so it must be on or below the $x$-axis for $-2 \leq x \leq 3$. Therefore the solution set is $[-2, 3]$.

### Exercises
Solve each inequality.

1. $x^2 + 2x < 0$  
   $\{x \mid x < 0\}$  
   \{x \mid x > 0\}

2. $2x^2 - 16 < 0$  
   $\{x \mid -4 < x < 4\}$  
   $\{x \mid x < -4 \text{ or } x > 4\}$

3. $0 < 6x - x^2 - 5$  
   $\{x \mid 1 < x < 5\}$  
   $\{x \mid x < 1 \text{ or } x > 5\}$

4. $c^2 - 2c + 1 < 0$  
   $\{c \mid c < 1 \text{ or } c > 1\}$  
   $\{c \mid -1 < c < 1\}$

5. $2m^2 - m - 1 < 0$  
   $\{m \mid -1 < m < 1\}$  
   $\{m \mid m < -1 \text{ or } m > 1\}$

6. $x^2 < -8$  
   $\emptyset$

7. $x^2 - 4x - 12 < 0$  
   $\{x \mid x < -2 \text{ or } x > 6\}$  
   $\{x \mid 2 < x < 6\}$

8. $x^2 + 9x + 14 < 0$  
   $\{x \mid x < -7 \text{ or } x > -2\}$  
   $\{x \mid -1 < x < 2\}$

9. $x^2 - 7x + 10 = 0$  
   $\{x \mid 2 < x < 5\}$  
   $\{x \mid 5 < x < 2\}$

10. $2x^2 + 3x - 3 < 0$  
    $\{x \mid -3 < x < 1\}$  
    $\{x \mid -3 < x < 1\}$

11. $11x^2 - 23x + 15 > 0$  
    $\{x \mid x < -2 \text{ or } x > 3\}$  
    $\{x \mid x < 2 \text{ or } x > 3\}$

12. $-6x^2 - 11x + 2 < 0$  
    $\{x \mid -2 < x < 1\}$  
    $\{x \mid -1 < x < 2\}$

13. $2x^2 - 11x + 12 > 0$  
    $\{x \mid x < 2 \text{ or } x > 6\}$  
    $\{x \mid x < 2 \text{ or } x > 6\}$

14. $14x^2 - 4x + 5 < 0$  
    $\{x \mid 1/3 < x < 5\}$  
    $\{x \mid 1/3 < x < 5\}$

15. $15x^2 - 16x + 5 > 0$  
    $\emptyset$  
    $\emptyset$

16. $-x^2 - 64 \leq -64$  
    $\{x \mid x \leq -16\}$  
    $\{x \mid x \leq -16\}$

17. $x^2 + 3x > 0$  
    $\{x \mid x < -3 \text{ or } x > 0\}$  
    $\{x \mid x < -3 \text{ or } x > 0\}$

18. $2x^2 + 2x > 4$  
    $\{x \mid x < -2 \text{ or } x > 1\}$  
    $\{x \mid x < -2 \text{ or } x > 1\}$

19. $9x^2 + 12x + 9 < 0$  
    $\emptyset$  
    $\emptyset$

20. $9x^2 + 12x + 9 < 0$  
    $\emptyset$  
    $\emptyset$

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### 4-8 Practice

#### Quadratic Inequalities

**Graph each inequality.**

1. \( y \leq x^2 + 4 \)
2. \( y > x^2 + 6x + 6 \)
3. \( y < 2x^2 - 4x - 2 \)

**Solve each inequality.**

4. \( x^2 + 2x + 1 > 0 \)  
   All reals
5. \( x^2 - 3x + 2 \leq 0 \)  
   \( \{x \mid -1 \leq x \leq 3\} \)
6. \( x^2 + 10x + 7 \geq 0 \)  
   \( \{x \mid x \leq 2 \text{ or } x \geq 5\} \)
7. \( x^2 - x - 20 > 0 \)  
   \( \{x \mid x < -4 \text{ or } x > 5\} \)
8. \( x^2 - 10x + 16 < 0 \)  
   \( \{x \mid 2 < x < 8\} \)
9. \( x^2 + 4x + 5 \leq 0 \)  
   \( \emptyset \)
10. \( x^2 + 14x + 49 \geq 0 \)  
   All reals
11. \( x^2 - 5x > 14 \)  
   \( \{x \mid x < -2 \text{ or } x > 7\} \)
12. \( -x^2 - 15 \leq 8x \)  
   \( \{x \mid -5 \leq x \leq -3\} \)
13. \( -x^2 + 5x - 7 \leq 0 \)  
   All reals
14. \( 9x^2 + 36x + 36 \leq 0 \)  
   \( \{x \mid x \leq -2\} \)
15. \( 9x \leq 12x^2 \)  
   \( \{x \mid x \leq 0 \text{ or } x \geq \frac{3}{4}\} \)
16. \( 4x^2 + 4x + 1 > 0 \)  
   \( \{x \mid x \neq -\frac{1}{2}\} \)
17. \( 5x^2 + 10 \geq 27x \)  
   \( \{x \mid x \leq \frac{2}{5} \text{ or } x \geq \frac{5}{3}\} \)
18. \( 9x^2 + 31x + 12 \leq 0 \)  
   \( \{x \mid -3 \leq x \leq \frac{4}{3}\} \)
19. **FENCING** Vanessa has 180 feet of fencing that she intends to use to build a rectangular play area for her dog. She wants the play area to enclose at least 1800 square feet. What are the possible widths of the play area? 30 ft to 60 ft

**20. BUSINESS** A bicycle maker sold 300 bicycles last year at a profit of $300 each. The maker wants to increase the profit margin this year, but predicts that each $20 increase in profit will reduce the number of bicycles sold by 10. How many $20 increases in profit can the maker add in and expect to make a total profit of at least $100,000? From 5 to 10

### 4-8 Word Problem Practice

#### Quadratic Inequalities

**1. HUTS** The space inside a hut is shaded in the graph. The parabola is described by the equation \( y = -\frac{5}{2}(x - 1)^2 + 4 \).

**2. DISCRIMINANTS** Consider the equation \( ax^2 + bx + c = 0 \). Assume that the discriminant is zero and that \( a \) is positive. What are the solutions of the inequality \( ax^2 + bx + c \leq 0 \)?

**3. KIOSKS** Caleb is designing a kiosk by wrapping a piece of sheet metal with dimensions \( x + 5 \) inches by \( 4x + 8 \) inches into a cylindrical shape. Ignoring cost, Caleb would like a kiosk that has a surface area of at least 4480 square inches. What values of \( x \) satisfy this condition?

(20) \( x > 30 \) (Note that the values of \( x \) result in a higher product, but negative lengths do not make sense.)

### Answers (Lesson 4.8)

- **4. DAMS** The Hoover Dam is a concrete arch dam designed to hold the water of Lake Mead. At its center, the dam’s height is approximately 725 feet, and the dam varies from 45 to 660 feet thick. The dark line on this sketch of the cross-section of the dam is a parabola.

**a.** Write an equation for the Hoover Dam parabola. Let the height be the y-value of the parabola and the thickness be the x-value of the parabola. (Hint: the equation will be in the form: \( y = ax^2 \) maximum thickness \( + \) maximum height)

**b.** Using your equation, graph the parabola of the Hoover Dam for 45 \( \leq x \leq \) 660.

**c.** Estimate to the nearest foot the thickness of the dam when the height is 200 feet.

353 feet
Graphing Absolute Value Inequalities

You can solve absolute value inequalities by graphing in much the same manner you graphed quadratic inequalities. Graph the related absolute function for each inequality by using a graphing calculator. For > and ≥, identify the x-values, if any, for which the graph lies below the x-axis. For < and ≤, identify the x-values, if any, for which the graph lies above the x-axis.

For each inequality, make a sketch of the related graph and find the solutions rounded to the nearest hundredth.

1. $|x - 3| > 0$
   - $x > 3$ or $x < 3$

2. $|x - 6| < 0$
   - $-6 < x < 6$

3. $-|x + 4| + 8 < 0$
   - $-12 < x < 4$

4. $|2x + 6| - 2 ≥ 0$
   - $x ≤ -7$ or $x ≥ -5$

5. $|3x - 3| ≥ 0$
   - all real numbers

6. $|x - 7| ≤ 5$
   - $2 < x < 12$

7. $|2x - 1| > 13$
   - $x < -7.1$ or $x > 2$

8. $|x - 3.6| ≤ 4.2$
   - $-0.6 ≤ x ≤ 7.8$

9. $|2x + 5| ≤ 7$
   - $-6 ≤ x ≤ 1$

Example 1

Solve $x^2 + x ≥ 6$.

Place the calculator in Dot mode. Enter the inequality into $Y_1$. Then trace the graph and describe the solution as an inequality.

Keystrokes:

1. $Y = x^2 + x - 6$
2. $Y_1 ≥ 0$
3. $|x| ≤ 2$ or $x ≥ 3$

Example 2

Solve $2x^2 + 4x - 5 ≤ 3$.

Place the left side of the inequality in $Y_1$ and the right side in $Y_2$. Determine the points of intersection. Use the intersection points to express the solution set of the inequality. Be sure to set the calculator to Connected mode.

Keystrokes:

1. $Y = 2x^2 + 4x - 5$
2. $Y_2 ≤ 3$

Press $\text{TRACE}$ (KALK) 5 and use the $\text{2nd}$ key to move the cursor to the left of the first intersection point. Press $\text{TRACE}$ then move the cursor to the right of the intersection point and press $\text{ENTER}$. One of the values used in the solution set is displayed. Repeat the procedure on the other intersection point.

The solution is $x ≤ -3.24$ or $x ≥ 1.24$.

Exercises

Solve each inequality.

1. $x^2 - 10x + 21 ≤ 0$
   - $(x - 7)(x - 3)$
   - $7 ≤ x ≤ 3$

2. $x^2 - 9 < 0$
   - $(x - 3)(x + 3)$
   - $-3 < x < 3$

3. $x^2 + 10x + 25 > 0$
   - $(x + 5)(x + 5)$
   - $x ≠ -5$

4. $x^2 + 3x ≤ 28$
   - $(x - 4)(x + 7)$
   - $-7 ≤ x ≤ 4$

5. $2x^2 + x ≥ 3$
   - $(2x - 3)(x + 1)$
   - $x ≥ -1.5$

6. $4x^2 + 12x + 9 > 0$
   - $(2x + 3)(2x + 3)$
   - $x ≠ -1.5$

7. $23 > x^2 - 10x$
   - $(x - 1)(x - 23)$
   - $x < 1$ or $x > 23$

8. $x^2 - 4x - 13 ≤ 0$
   - $(x - 7)(x + 1)$
   - $-1 ≤ x ≤ 7$

9. $(x - 1)(x - 3) > 0$
   - $(x - 1)(x - 3)$
   - $x < 1$ or $x > 3$
Chapter 4 Assessment Answer Key

Quiz 1 (Lessons 4-1 and 4-2)
Page 59

1. \(-3; x = -1; -1\)

2. \(f(x) = x^2 + 2x - 3\)

3. \(3, -1\)

4. \(\frac{3}{2}\) between 1 and 2; between \(-6\) and \(-5\)

5. \(-96; 2\) complex roots

Quiz 2 (Lessons 4-3 and 4-4)
Page 59

1. \(\{-5, \frac{2}{3}\}\)

2. \(\{-9, 5\}\)

3. \(\frac{3}{2}\)

4. \(x^2 + 4x - 12 = 0\)

5. \(3x^2 + 10x - 8 = 0\)

6. \(4i\sqrt{5}\)

7. \(-6\sqrt{2}\)

8. \(-11 + 3i\)

9. \(68 + 4i\)

10. \(\frac{1}{2} + \frac{1}{2}i\)

Quiz 3 (Lessons 4-5 and 4-6)
Page 60

1. \(\{-10, 2\}\)

2. \(1 \pm 3\sqrt{5}\)

3. \(B\)

4. \(2 \pm \sqrt{5}\)

5. \(-96; 2\) complex roots

Quiz 4 (Lessons 4-7 and 4-8)
Page 60

1. \(C\)

2. \(\{0, \frac{1}{4}\}\)

3. \(\{-2, 9\}\)

4. \(\frac{25}{34} + \frac{15i}{34}\)

Mid-Chapter Test
Page 61

1. \(B\)

2. \(H\)

3. \(A\)

4. \(F\)

5. \(D\)

6. \(1, 3\)

7. \(\{-9 \frac{1}{2}\}\)

8. \(\{-2, 9\}\)

9. \(\frac{25}{34} + \frac{15i}{34}\)
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<td>2. G</td>
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<td>12. J</td>
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<tr>
<td>1. false; complex conjugates</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2. false; constant term</td>
<td></td>
<td></td>
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</tr>
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<td>3. false; quadratic inequality</td>
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<tr>
<td>4. false; roots</td>
<td></td>
<td></td>
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<td>5. true</td>
<td></td>
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<tr>
<td>6. false; minimum value</td>
<td></td>
<td></td>
<td></td>
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<td>7. false; quadratic term</td>
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<tr>
<td>8. true</td>
<td></td>
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<tr>
<td>9. true</td>
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<tr>
<td>10. false; discriminant</td>
<td>Sample answer: The value that determines the roots of a quadratic equation.</td>
<td></td>
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</tr>
<tr>
<td>11. Sample answer: a quadratic equation written as ( ax^2 + bx + c = 0 )</td>
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<tr>
<td>12. B: 1 and 7; 14</td>
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</tr>
</tbody>
</table>
Chapter 4 Assessment Answer Key

Form 2A  Form 2B
Page 65  Page 66  Page 67  Page 68

1. B  11. C

2. G
2. H

3. A
3. B
13. C
13. D
14. J
14. F

4. H
4. J
15. D
15. A

5. D
6. G
5. A
6. H
17. B
17. D

7. C
8. F
8. J
18. H

9. D
9. A
19. C
19. A

10. F
10. F

20. J
Sample answer:
B: \(16x^2 + 3 = 0\)

20. F
Sample answer:
B: \(9x^2 + 2 = 0\)
Chapter 4 Assessment Answer Key

Form 2C

Page 69

1. \( f(x) = -5x^2 + 10x \)

2. **maximum; 4**

3. **2, 4**

4. **\left\{ -3, \frac{2}{5} \right\}**

5. **9 in. by 16 in.**

6. **6 + 12j ohms**

7. **\frac{37}{17} - \frac{22}{17} j amps**

8. **4x^2 + 21x - 18 = 0**

9. **\left\{ -8, 2 \right\}**

10. \( \left\{ -2 \pm \sqrt{13} \right\} \)

11. \( \left\{ -2, \frac{1}{2} \right\} \)

12. **\frac{3 \pm i\sqrt{31}}{10}**

13. **0; 1 real, rational root**

14. **33; 2 real, irrational roots**

15. \( x = -5; \) down

16. \( y = \frac{3}{2}(x - 2)^2 - 1 \)

17. \( y = (x - 3)^2 - 1 \)

18. \( h(t) = -16(t - 1.5)^2 + 51; 51\text{ft} \)

19. \( \left\{ x \mid x \leq -\frac{1}{2} \text{ or } x \geq 3 \right\} \)

20. **B: 9x^2 - 7 = 0**

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Chapter 4 Assessment Answer Key

Form 2D
Page 71

1. $f(x) = x^2 - 4x + 3$  
   Minimum: $x = 2$  
   $f(2) = 3$

2. Minimum; $-17$

3. $1, -3$

4. $\left\{ -1, \frac{4}{3} \right\}$

5. 8 in. by 18 in.

6. $9 - 6j$ ohms

7. $\frac{23}{17} - \frac{27}{17}j$ ohms

8. $2x^2 + 5x - 12 = 0$

Page 72

9. $\left\{ -\frac{2 + \sqrt{6}}{3} \right\}$

10. $\left\{ 4 + \sqrt{2} \right\}$

11. $\left\{ -1, \frac{2}{3} \right\}$

12. $\left\{ \frac{9 + \sqrt{41}}{4} \right\}$

13. 0; 1 real, rational root

14. $-8$; 2 complex roots

15. $(6, -5)$; $x = 6$; down

16. $y = \frac{1}{4}(x + 4)^2 + 2$

17. $y = (x + 2)^2 + 4$

18. $h(t) = -16(t - 2)^2 + 76$; 76 ft

19. $\left\{ x \mid -\frac{3}{2} < x < 5 \right\}$

20. $16x^2 - 5 = 0$

B: $16x^2 - 5 = 0$
Chapter 4 Assessment Answer Key

Form 3
Page 73

1. $f(x) = 3x^2 + 2x + 3$

minimum; $\frac{22}{25}$

2. $\{0.35, 0.85\}$

3. $-2 < k < 2$

4. $3, 6$

5. $\left\{\frac{1}{2}, \frac{5}{3}\right\}$

6. $12x^2 - 13x - 14 = 0$

7. $4 - 6i$

8. $\frac{1}{9} - \frac{4\sqrt{5}}{9}i$

9. $6 \pm 4\sqrt{2}$

10. $\left\{\frac{5 \pm i\sqrt{39}}{8}\right\}$

11. $\{3.5, 1\}$

12. $\{\frac{5}{2}, -\frac{1}{2}\}; x = -\frac{7}{2};$ down

13. $-3 < x \leq -2$ or $x = 1$

14. $y = -\frac{3}{5}(x + \frac{7}{2})^2 - \frac{1}{2}$

15. $y = -\frac{29}{200}(x + 9)^2 + \frac{29}{2}$

16. $\{x \mid x \leq -\frac{7}{2} \text{ or } x = 1\}$

17. $h(t) = -9.1(t - 32.5)^2 + 30,000; 30,000 \text{ ft}$

18. $16x^2 + 24x + 29 = 0$
<table>
<thead>
<tr>
<th>Score</th>
<th>General Description</th>
<th>Specific Criteria</th>
</tr>
</thead>
</table>
|      | **Superior**       | Shows thorough understanding of the concepts of graphing, analyzing, and finding the maximum and minimum values of quadratic functions; solving quadratic equations; computing with complex numbers; and solving inequalities.  
• Uses appropriate strategies to solve problems.  
• Computations are correct.  
• Written explanations are exemplary.  
• Goes beyond requirements of some of or all problems. |
| 4    | **Satisfactory**   | Shows an understanding of the concepts of graphing, analyzing, and finding the maximum and minimum values of quadratic functions; solving quadratic equations; computing with complex numbers; and solving inequalities.  
• Uses appropriate strategies to solve problems.  
• Computations are mostly correct.  
• Written explanations are effective.  
• Satisfies all requirements of problems. |
| 3    | **Nearly Satisfactory** | Shows an understanding of most of the concepts of graphing, analyzing, and finding the maximum and minimum values of quadratic functions; solving quadratic equations; computing with complex numbers; and solving inequalities.  
• May not use appropriate strategies to solve problems.  
• Computations are mostly correct.  
• Written explanations are satisfactory.  
• Satisfies the requirements of most of the problems. |
| 2    |                     | Final computation is correct.  
• No written explanations or work is shown to substantiate the final computation.  
• Satisfies minimal requirements of some of the problems. |
| 1    | **Nearly Unsatisfactory** | Shows little or no understanding of most of the concepts of graphing, analyzing, and finding the maximum and minimum values of quadratic functions; solving quadratic equations; computing with complex numbers; and solving inequalities.  
• Does not use appropriate strategies to solve problems.  
• Computations are incorrect.  
• Written explanations are unsatisfactory.  
• Does not satisfy the requirements of problems.  
• No answer may be given. |
| 0    | **Unsatisfactory** |                                                                 |
Chapter 4 Assessment Answer Key

Page 75, Extended-Response Test
Sample Answers

In addition to the scoring rubric found on page A33, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1. Student responses should indicate that using the Square Root Property, as Mi-Ling’s group did, would take less time than the other method since the equation is already set up as a perfect square set equal to a constant. To solve using the other method, the binomial would need to be expanded and the constant on the right brought to the left side of the equal sign.

2a. Jocelyn had trouble because the problem is impossible. No such parabola exists.

2b. Student responses will vary. One of the three conditions must be omitted or modified. Sample answer: “...that passes through (−1, −12).”

2c. Answers will vary and depend on the answer for part b. For example, for the sample answer in part b above, a possible equation is: $y = −2(x + 3)^2 − 4$.

3a. Answer must be of the form $y = a(x − h)^2 + 8$ where $h$ is any real number and $a < 0$.

3b. Answers must be of the form $y = a[x − (h + n)]^2 + 8$ where $h$ and $a$ represent the same values as in part a. The student choice is for the value of $n$. The student should indicate that the graph will shift to the left $n$ units if his or her value of $n$ is negative, but will shift the graph to the right $n$ units if the chosen value of $n$ is positive.

4. Students should indicate that Joseph’s answer is not correct. In Step 2, when he completed the square by adding 9 inside the parentheses, he actually added $2(9) = 18$ to the right side of the equation, so he must subtract 18 from the constant on the same side, rather than add 9, to keep the statements equivalent. The correct solution is $f(x) = 2(x + 3)^2 − 23$.

5a. > The graph is strictly above the $x$-axis for all values of $x$ other than 2.

5b. <; The graph is never below the $x$-axis.

5c. ≥; The graph is always on or above the $x$-axis.
Chapter 4 Assessment Answer Key

Standardized Test Practice
Page 76

1. ○ ○ ○ ○

2. ○ ○ ○ ○

3. ○ ○ ○ ○

4. ○ ○ ○ ○

5. ○ ○ ○ ○

6. ○ ○ ○ ○

7. ○ ○ ○ ○

8. ○ ○ ○ ○

9. ○ ○ ○ ○

10. ○ ○ ○ ○

11. ○ ○ ○ ○

12. ○ ○ ○ ○

13. ○ ○ ○ ○

14. ○ ○ ○ ○

15. 

16. 

Chapter 4 A35 Glencoe Algebra 2
17. 17
18. \(-3 \ \frac{3}{4}\)

19. inconsistent

20. \((-2, 0), (-2, 8), (0, -2), (8, -2)\)

21. 92

22. (2, -3)

23. \(\pm 2i\sqrt{5}\)

24. 136 ft; 1.5 s

25. -1, 3

26. \((-2, 3)\)

27. -3, 2 complex roots

28a. \(y = (x - \frac{7}{2})^2 - \frac{29}{4}\)

28b. \(\left(\frac{7}{2}, -\frac{29}{4}\right)\)

28c. \(x = \frac{7}{2}\)